UNIT- III

Dr.V.Vijaya Kishore, M.Tech., Ph.D.
PROFESSOR
Department of ECE
G.Pullaiah College of Engg. & Technology
KURNOOL.

A DTS is a system that accepts discrete i/p and produces discrete o/p. A DTS is Mathematically represented by difference equation. Physically a DTS is realized or implemented either as a digital H/W (Like MP/µc) or S/W running on digital H/W (Like pc)

Processing of DTsignal by digital H/W involves Mathematical operations like addition, Multiplication and delay elements. All calculations are performed by using fixed point arithmetic or floating point arithmetic. Time taken to process DT signal and Computational Complexity depends on no of Calculations involved and type of arithmetic operation used for computation. These issues are addressed in structures of realization of DTS.

* DT IIR System : Let H(z) is Transfer f^n of IIR system

General form of transfer function of IIR sym is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\text{o/p of DTS in z-domain}}{\text{i/p of DTS in z-domain}}$$

$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

$\Rightarrow$

$$Y(z)\left(1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}\right) = X(z)\left(b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}\right)$$

applying inverse Z-T on both sides

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + \dots + a_N y(n-N) =$$
$$b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_M x(n-M)$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \dots - a_N y(n-N) + b_0 x(n) + b_1 x(n-1)$$
$$+ b_2 x(n-2) + \dots + b_M x(n-M)$$

$\Rightarrow$

$$\boxed{y(n) = -\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m)}$$

o/p depends on past o/ps so IIR systems are Recursive systems

DT FIR System: $H(z)$ is Transfer f^n

General form $H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}$

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}$$

$$Y(z) = \left[b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}\right] X(z)$$

Taking IZT on both sides.

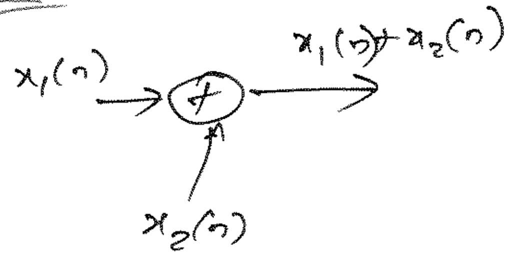
$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_{N-1} x(n-(N-1))$$

$$\boxed{y(n) = \sum_{m=0}^{N-1} b_m \cdot x(n-m)}$$

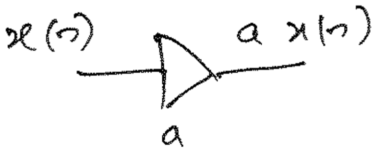
o/p at any time 'n' depends only on present & past i/ps so FIR systems are Non-recursive Systems.

* Basic Elements of Realization

1. Adder



2. Const. multiplier



3. Unit delay element

4. Unit advance

* Structures for Realization of IIR System

General time domain form of IIR System of N^{th} order is given by

$$y(n) = -\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m)$$

In Z domain N^{th} order IIR system is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

M - No of zeros ; N - No of poles.

Different structures are used for realizing IIR system

1. Direct form - I
2. Direct form - II
3. Cascade form
4. Parallel form

1. Direct form - I

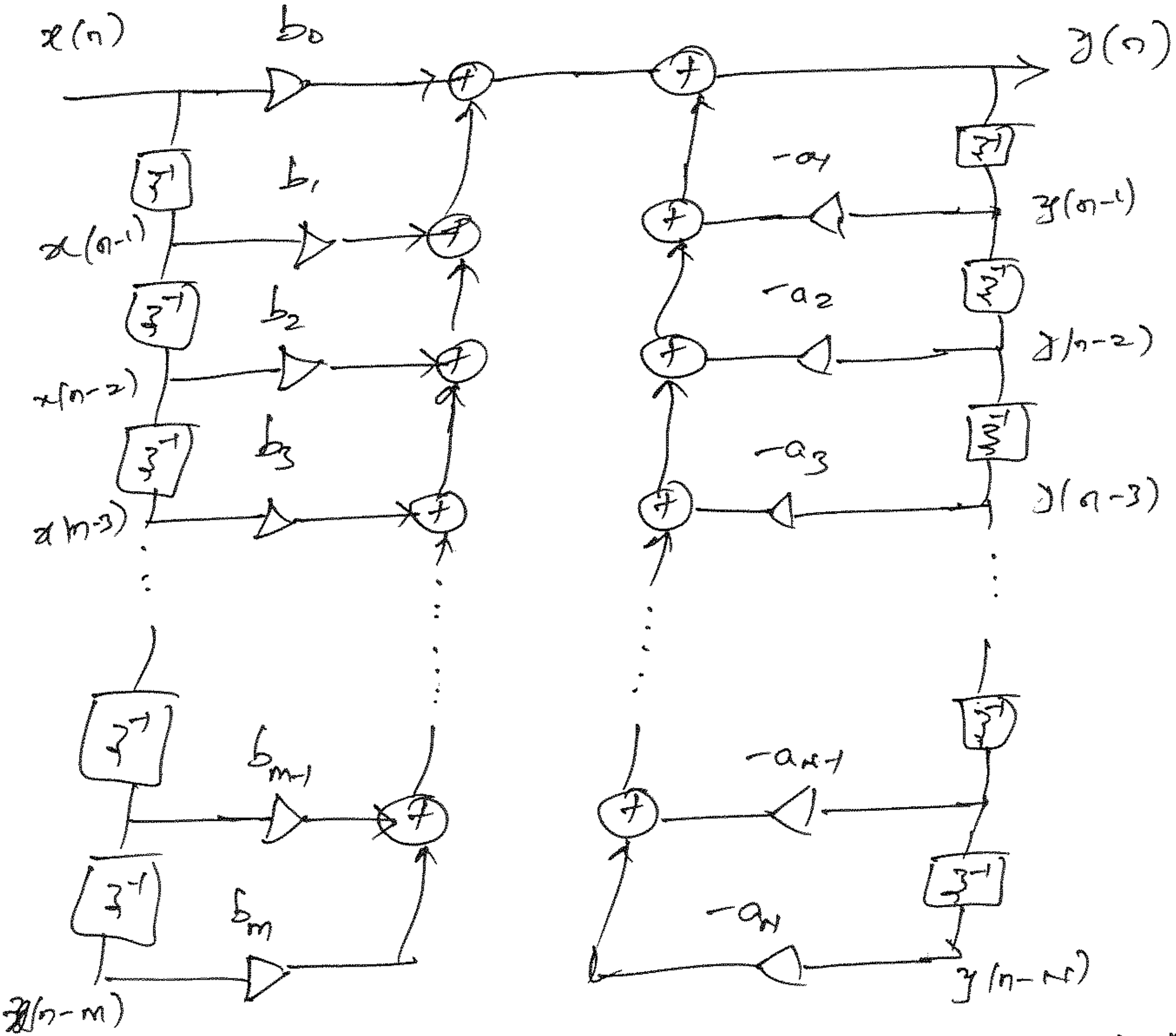
$$y(n) = -\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m)$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \dots a_N y(n-N) + b_0 x(n)$$
$$+ b_1 x(n-1) + b_2 x(n-2) + \dots + b_M x(n-M)$$

taking Z Transform

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - \dots a_N z^{-N} Y(z) + b_0 X(z)$$
$$+ b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \dots + b_M z^{-M} X(z)$$

Direct form-I structure can be realized as shown below.



Direct form-I structure is realization of N^{th} order discrete time s/m with 'M' zeros and 'N' poles. It involves

No of multiplication : $M+N+1$

No of Additions : $M+N$

No of delays : $M+N$

No of Memory locations to store delay signals : $M+N$

When no of delays in a structure is equal to the order of system, the structure is called Canonic structure.

In direct form-I structure the no of delays is not equal to order of system and so direct form-I structure is Non-canonic structure

2. Direct form-II of IIR System

$$y(n) = -\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m)$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - a_3 y(n-3) - \dots - a_N y(n-N)$$
$$+ b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_M x(n-M)$$

applying z.T

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

let $$H(z) = \frac{Y(z)}{X(z)} = \frac{W(z)}{X(z)} \times \frac{Y(z)}{W(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

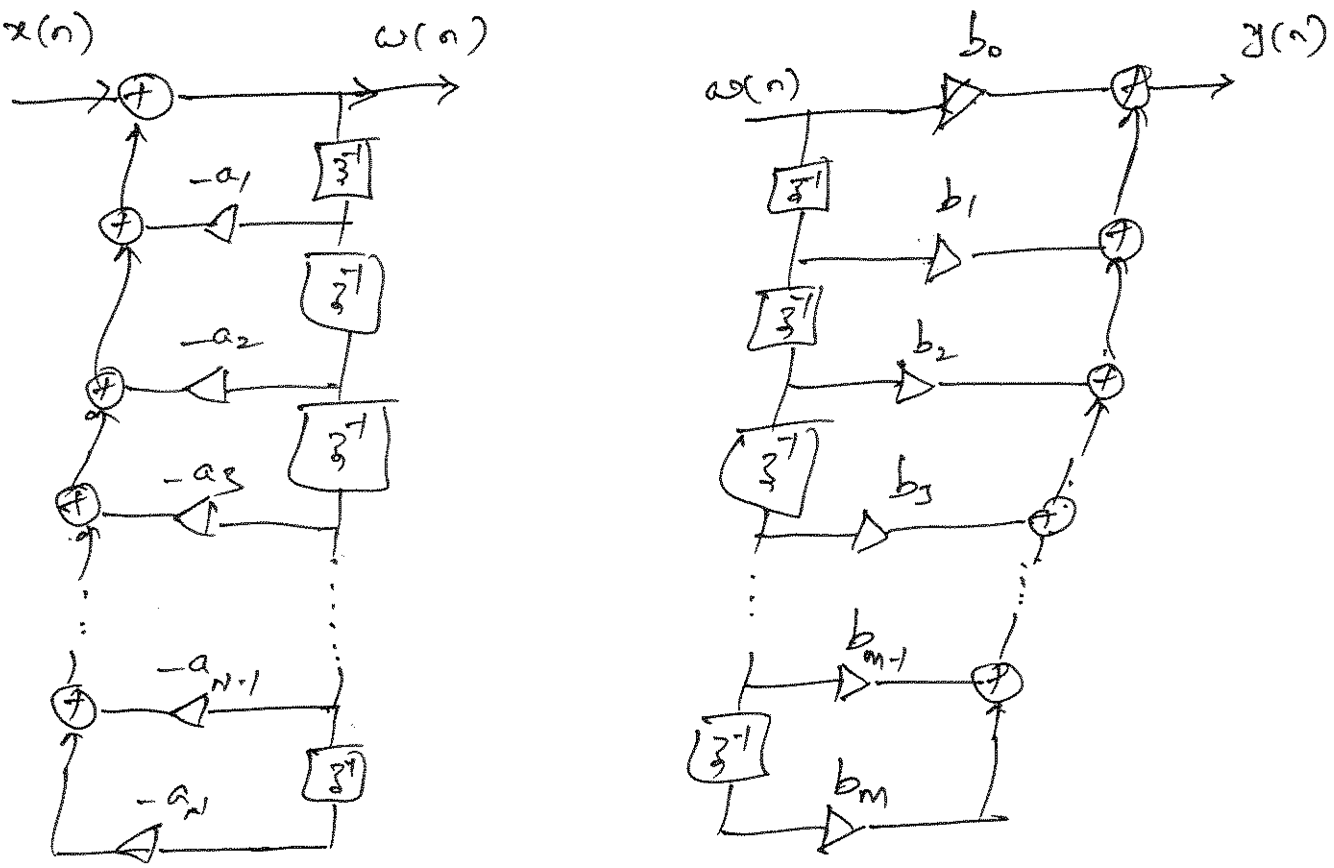
$$\frac{Y(z)}{W(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}$$

Applying IZT for above two equations.

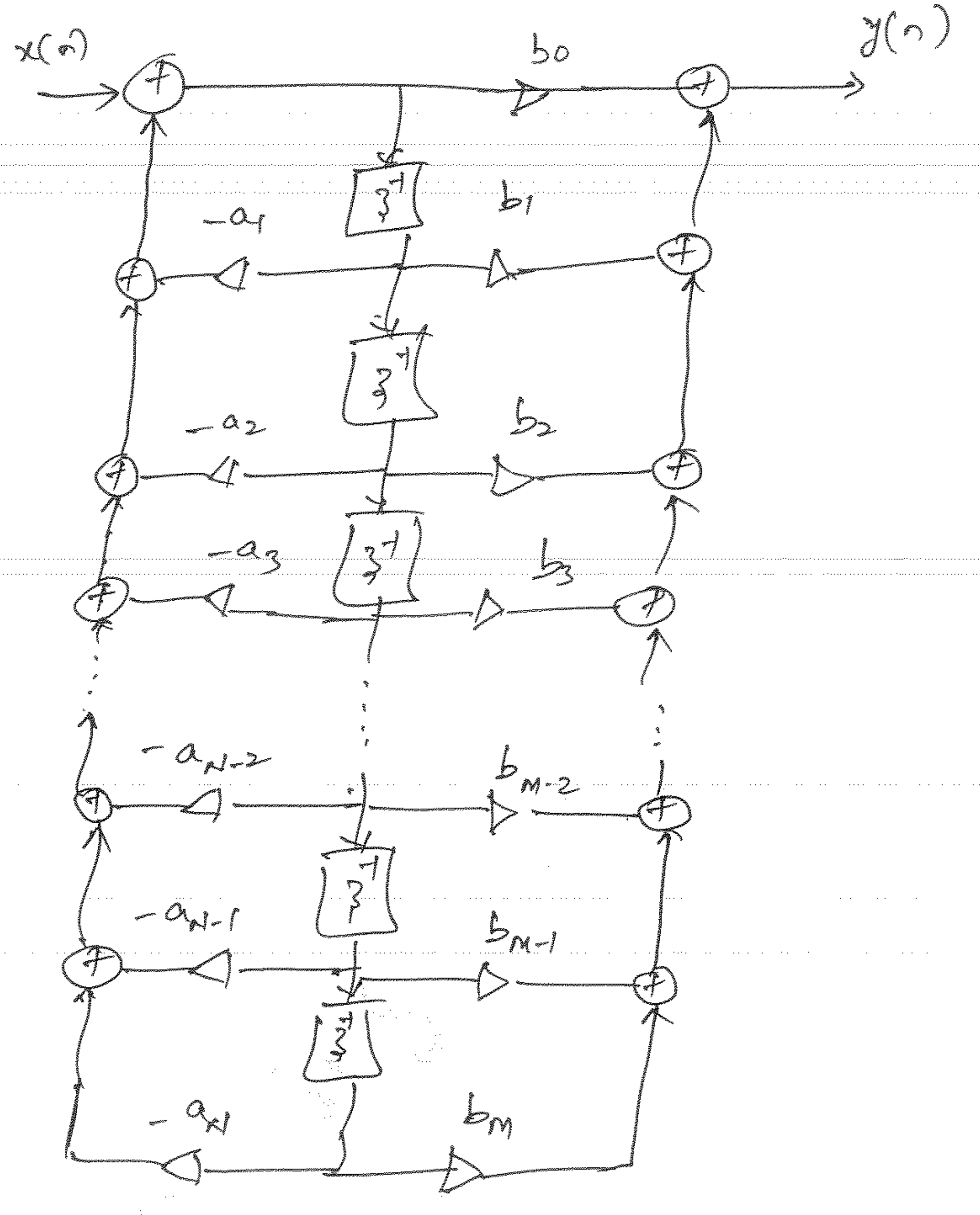
$$w(n) = -a_1 w(n-1) - a_2 w(n-2) - \dots - a_N w(n-N) + x(n) \quad \text{— (A)}$$

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \dots + b_M w(n-M) \quad \text{— (B)}$$

Eq (A) is realized as. Eq (B) as shown below



overall direct form II structure is shown below



Here No of delays is equal to order of system and hence direct form-II is Canonic structure

Direct form-II is realization of N^{th} order DTS with 'M' zero's and 'N' poles. It involves

No of multiplication : $M+N+1$

No of additions : $M+N$

$N \geq M$, No of delays is equal to N, so 'N' memory locations required

3. Cascade form Realization of IIR

$H(z)$ is expressed as product of number of second order or first order sections.

$$H(z) = \frac{Y(z)}{X(z)} = H_1(z) \times H_2(z) \times \dots \times H_m(z)$$

$$= \prod_{i=1}^{m} H_i(z)$$

$$H_i(z) = \frac{C_{0i} + C_{1i} z^{-1} + C_{2i} z^{-2}}{d_{0i} + d_{1i} z^{-1} + d_{2i} z^{-2}} \quad (2^{nd} \text{ order section})$$

$$H_i(z) = \frac{C_{0i} + C_{1i} z^{-1}}{d_{0i} + d_{1i} z^{-1}} \quad (1^{st} \text{ order section})$$

$$X(z) \longrightarrow \boxed{H_1(z)} - \boxed{H_2(z)} - \boxed{H_3(z)} - \cdots \boxed{H_m(z)} \longrightarrow Y(z)$$

Difficulty in cascade are

i) Decision of pairing poles and zeros
ii) Deciding order of cascading (1^{st} & 2^{nd} order)
iii) Scaling multipliers should be provided b/n individual sections to prevent system variables from becoming too large or too small.

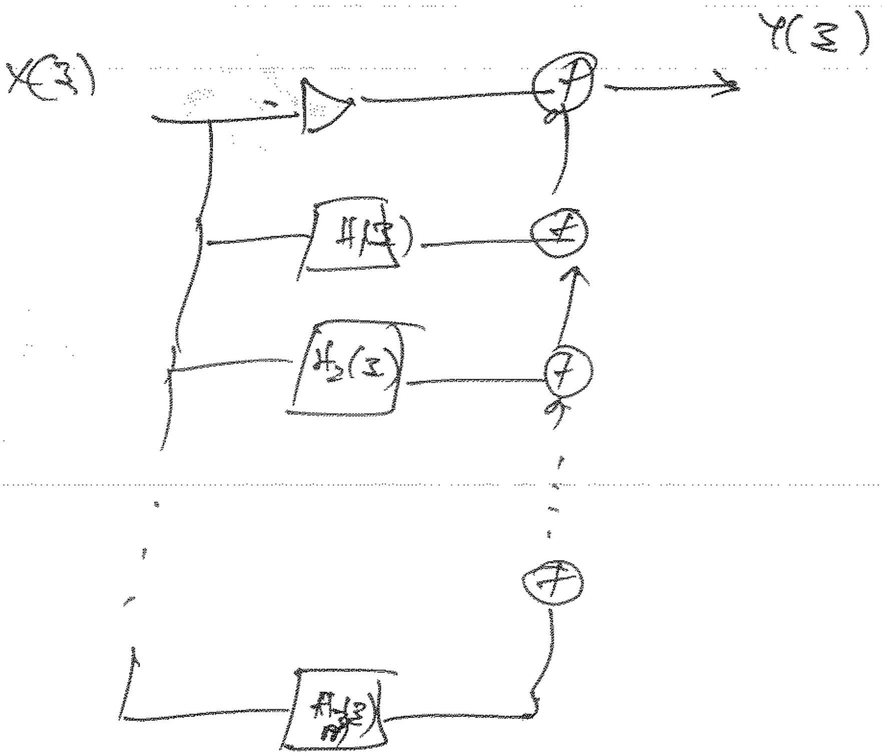
6. Parallel form Realization of IIR system

$$H(z) = \frac{Y(z)}{X(z)} = C + H_1(z) + H_2(z) + \dots + H_m(z)$$

$$= C + \sum_{i=1}^{m} H_i(z)$$

$$H_i(z) = \frac{C_{0i} + C_{1i} z^{-1}}{d_{0i} + d_{1i} z^{-1} + d_{2i} z^{-2}} \quad (2^{nd} \text{ order})$$

$$H_i(z) = \frac{C_{0i}}{d_{0i} + d_{1i} z^{-1}} \quad (1^{st} \text{ order})$$



Prob: obtain direct form-I, direct form-II, cascade form and parallel form realizations of LTI system described by $y(n) = -\frac{3}{8} y(n-1) + \frac{3}{32} y(n-2) + \frac{1}{64} y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$

Sol^n :- Given $y(n) = -\frac{3}{8} y(n-1) + \frac{3}{32} y(n-2) + \frac{1}{64} y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$

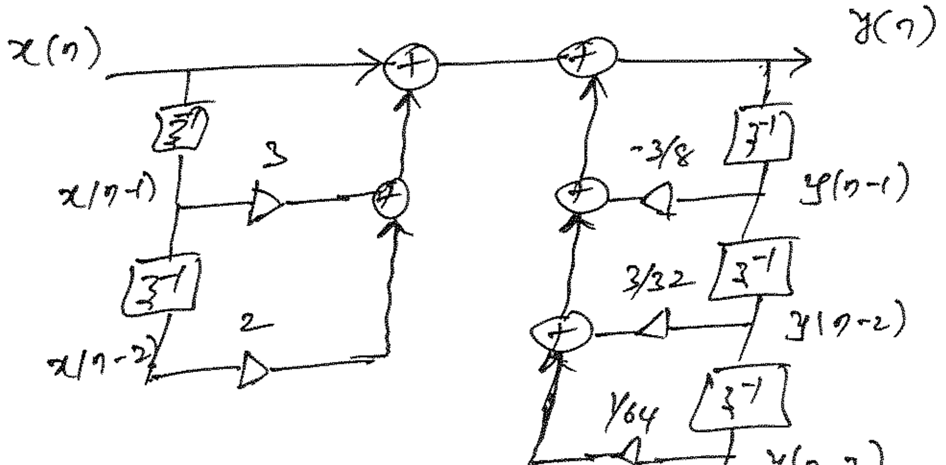
applying z-T on both sides.

$$Y(z) = -\frac{3}{8} \cdot z^{-1} Y(z) + \frac{3}{32} z^{-2} Y(z) + \frac{1}{64} z^{-3} Y(z) + X(z) + 3 z^{-1} X(z) + 2 \cdot z^{-2} X(z)$$

Transfer f^n $H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + \frac{3}{8} z^{-1} - \frac{3}{32} z^{-2} - \frac{1}{64} z^{-3}}$

Direct form-I structure

$$y(n) = -\frac{3}{8} y(n-1) + \frac{3}{32} y(n-2) + \frac{1}{64} y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$$



Direct form - II structure

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+3z^{-1}+2z^{-2}}{1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}}$$

$$H(z) = \frac{Y(z)}{X(z)} \times \frac{W(z)}{W(z)} = \frac{W(z)}{X(z)} \times \frac{Y(z)}{W(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}}$$

$$\frac{Y(z)}{W(z)} = 1+3z^{-1}+2z^{-2}$$

Consider $\frac{W(z)}{X(z)} = \frac{1}{1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}}$

$$W(z)\left[1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}\right] = X(z)$$

apply IZT on both sides

$$w(n)+\frac{3}{8}w(n-1)-\frac{3}{32}w(n-2)-\frac{1}{64}w(n-3) = x(n)$$

$$\Rightarrow \quad w(n) = -\frac{3}{8}w(n-1)+\frac{3}{32}w(n-2)+\frac{1}{64}w(n-3)+x(n) \quad -(1)$$

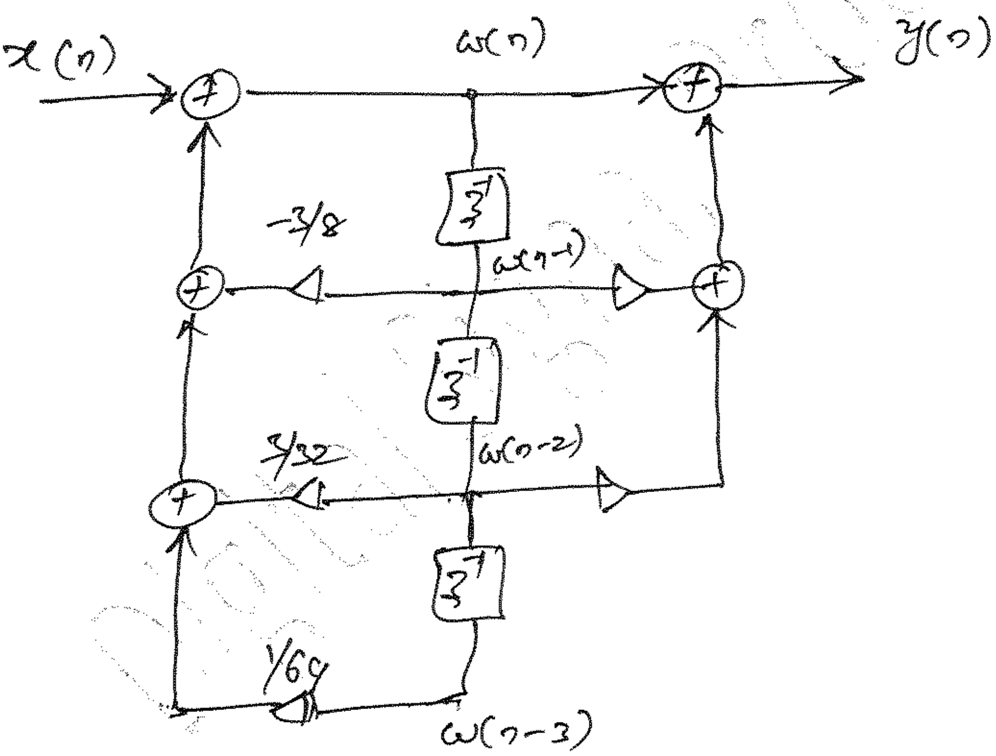
Consider $\dfrac{Y(z)}{W(z)} = 1 + 3z^{-1} + 2z^{-2}$

$$Y(z) = W(z)\left(1 + 3z^{-1} + 2z^{-2}\right)$$

apply IZT on both sides.

$$y(n) = w(n) + 3w(n-1) + 2w(n-2) \longleftarrow (2)$$

Using equations (1) & (2) direct form-II structure is shown below.



Cascade form : $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1 + 3z^{-1} + 2z^{-2}}{1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}}$

Numerator & denominator polynomials must be expressed in factored form.

Consider numerator polynomial:

$$1 + 3z^{-1} + 2z^{-2} = z^{-2}\left(z^2 + 3z + 2\right)$$

$$= z^{-2}(z+1)(z+2)$$

$$= \left(1+z^{-1}\right)\left(1+2z^{-1}\right)$$

Consider denominator polynomial

$$1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3} = z^{-3}\left(z^3 + \frac{3}{8}z^2 - \frac{3}{32}z - \frac{1}{64}\right)$$

$$= z^{-3}\left(z+\frac{1}{8}\right)\left(z^2 + \frac{2}{8}z - \frac{8}{64}\right)$$

Roots of $z^2 + \frac{2}{8}z - \frac{8}{64} = 0$ are

$$z = \frac{-\frac{2}{8} \pm \sqrt{\left(\frac{2}{8}\right)^2 - 4\left(\frac{-8}{64}\right)}}{2} \Rightarrow z = \frac{1}{4}, -\frac{1}{2}$$

$$\Rightarrow 1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3} = z^{-3}\left(z+\frac{1}{8}\right)\left(z-\frac{1}{4}\right)\left(z+\frac{1}{2}\right)$$

$$= z^{-1}\left(z+\frac{1}{8}\right)z^{-1}\left(z-\frac{1}{4}\right)z^{-1}\left(z+\frac{1}{2}\right)$$

$$= \left(1+\frac{1}{8}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)$$

$$\therefore H(z) = \frac{\left(1+z^{-1}\right)\left(1+2z^{-1}\right)}{\left(1+\frac{1}{8}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

We can write $H(z)$ as three first order factors

$$H(z) = \frac{1+z^{-1}}{1+\frac{1}{8}z^{-1}} \times \frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}} \times \frac{1}{1+\frac{1}{4}z^{-1}}$$

$$= H_1(z) \times H_2(z) \times H_3(z)$$

$$H_1(z) = \frac{1+z^{-1}}{1+\frac{1}{8}z^{-1}} \; ; \; H_2(z) = \frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}} \; ; \; H_3(z) = \frac{1}{1+\frac{1}{4}z^{-1}}$$

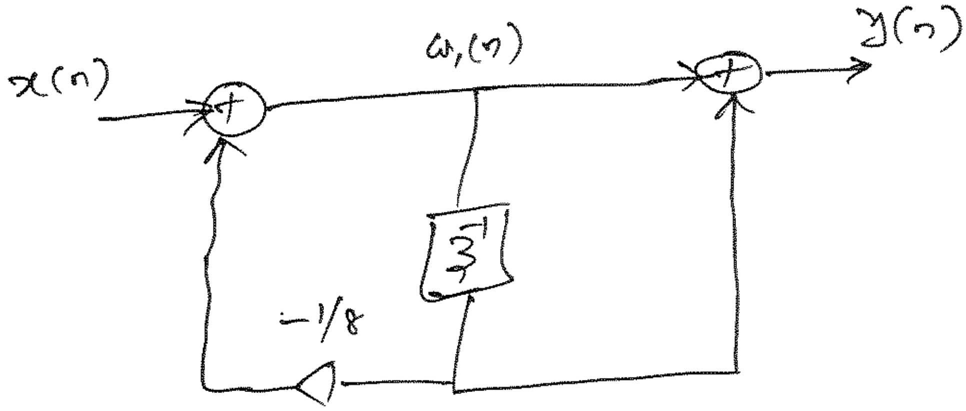
Express $H_1(z)$, $H_2(z)$ & $H_3(z)$ in Direct form-II structure

Consider $H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{1+z^{-1}}{1+\frac{1}{8}z^{-1}}$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W_1(z)}{X(z)} \times \frac{Y(z)}{W_1(z)} = \frac{1+z^{-1}}{1+\frac{1}{8}z^{-1}}$$

$$\frac{W_1(z)}{X(z)} = \frac{1}{1+\frac{1}{8}z^{-1}} \qquad \frac{Y(z)}{W_1(z)} = 1+z^{-1}$$

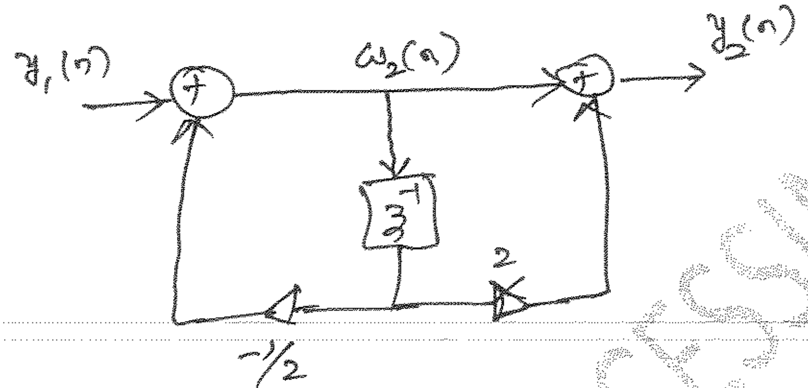
$$w_1(n) = -\frac{1}{8}w(n-1) + x(n) \quad ; \quad y(n) = w_1(n) + w_1(n-1)$$



Consider $H_2(z) = \frac{Y_2(z)}{Y_1(z)} = \frac{W_2(z)}{Y_1(z)} \times \frac{Y_2(z)}{W_2(z)} = \frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}}$

$$\frac{W_2(z)}{Y_1(z)} = \frac{1}{1+\frac{1}{2}z^{-1}} \qquad \frac{Y_2(z)}{W_2(z)} = 1+2z^{-1}$$

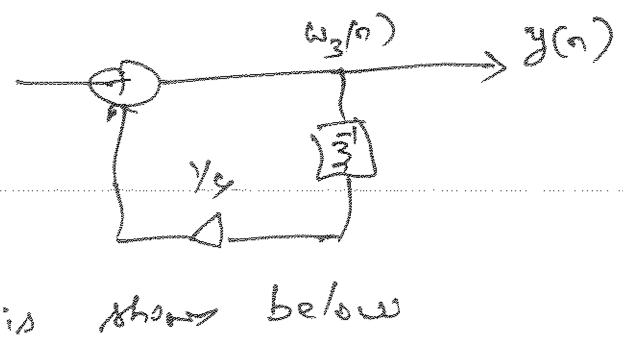
$$w_2(n) = -\frac{1}{2}w_2(n-1) + y_1(n) \quad ; \quad y_2(n) = w_2(n) + 2w_2(n-1)$$



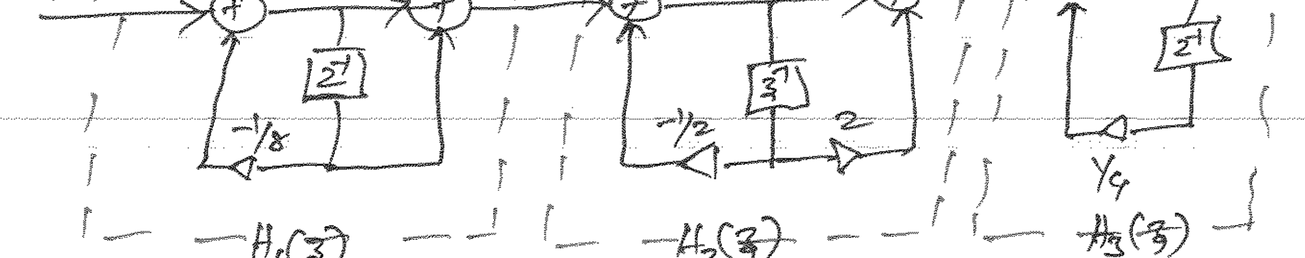
Consider $H_3(z) = \frac{Y(z)}{Y_2(z)} = \frac{W_3(z)}{Y_2(z)} \times \frac{Y(z)}{W_3(z)} = \frac{1}{1-\frac{1}{4}z^{-1}}$

$$\frac{W_3(z)}{Y_2(z)} = \frac{1}{1-\frac{1}{4}z^{-1}} \qquad \frac{Y(z)}{W_3(z)} = \frac{1}{1}$$

$$w_3(n) = \frac{1}{4}w_3(n-1) + y_2(n) \quad ; \quad y(n) = w_3(n)$$



Cascaded structure is shown below



Parallel form :
$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$$H(z) = \frac{A}{1+\frac{1}{8}z^{-1}} + \frac{B}{1+\frac{1}{2}z^{-1}} + \frac{C}{1-\frac{1}{4}z^{-1}} = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$$A = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \times (1+\tfrac{1}{8}z^{-1})\Bigg|_{z^{-1}=-8} = \frac{(1-8)(1-16)}{(1-4)(1+2)} = \frac{-105}{9} = \frac{-35}{3}$$

$$B = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \times (1+\tfrac{1}{2}z^{-1})\Bigg|_{z^{-1}=-2} = \frac{(1-2)(1-4)}{(1-\frac{1}{4})(1+\frac{1}{2})} = \frac{(-1)(-3)}{\frac{3}{4}\times\frac{3}{2}} = \frac{8}{3}$$

$$C = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \times (1-\tfrac{1}{4}z^{-1})\Bigg|_{z^{-1}=4} = \frac{(1+4)(1+8)}{(1+\frac{1}{2})(1+2)} = \frac{5\times 9}{\frac{3}{2}\times 3} = 10$$

$$\therefore \; H(z) = \frac{-35/3}{1+\frac{1}{8}z^{-1}} + \frac{8/3}{1+\frac{1}{2}z^{-1}} + \frac{10}{1-\frac{1}{4}z^{-1}}$$

$$= H_1(z) + H_2(z) + H_3(z)$$

Dr.V.Vijaya Kishore,M.Tech.,Ph.D.
PROFESSOR
Department of ECE
G.Pullaiah College of Engg. & Technology
KURNOOL.

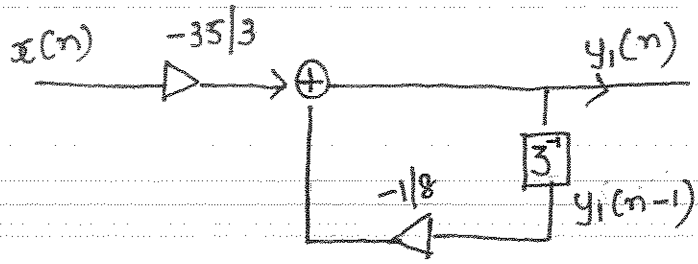
$$H_1(z) = \frac{-35/3}{1+\frac{1}{8}z^{-1}} = \frac{Y_1(z)}{X(z)}$$

$$\frac{-35}{3}X(z) = Y_1(z)\left[1+\frac{1}{8}z^{-1}\right]$$

Apply I.Z.T & shifting property

$$\frac{-35}{3}x(n) = y_1(n) + \frac{1}{8}y_1(n-1)$$

$$y_1(n) = \frac{-35}{3}x(n) - \frac{1}{8}y_1(n-1)$$



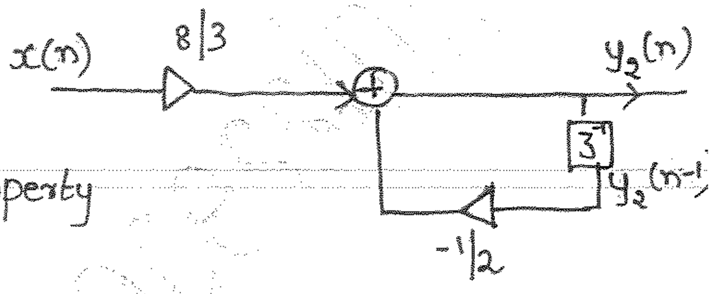
$$H_2(z) = \frac{8/3}{1+\frac{1}{2}z^{-1}} = \frac{Y_2(z)}{X(z)}$$

$$\frac{8}{3}X(z) = Y_2(z)\left[1+\frac{1}{2}z^{-1}\right]$$

$\frac{8}{3}$ ⊃ Apply I.Z.T & shifting property

$$\frac{8}{3}x(n) = y_2(n) + \frac{1}{2}y_2(n-1)$$

$$y_2(n) = \frac{8}{3}x(n) - \frac{1}{2}y_2(n-1)$$



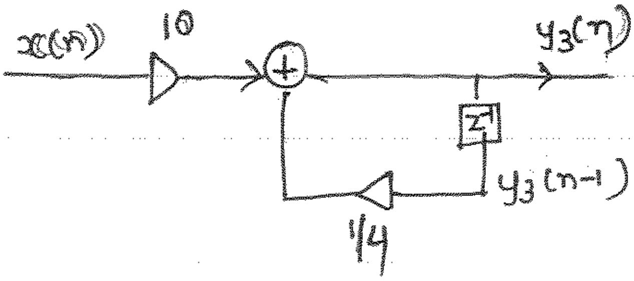
$$H_3(z) = \frac{10}{\left(1-\frac{1}{4}z^{-1}\right)} = \frac{Y_3(z)}{X(z)}$$

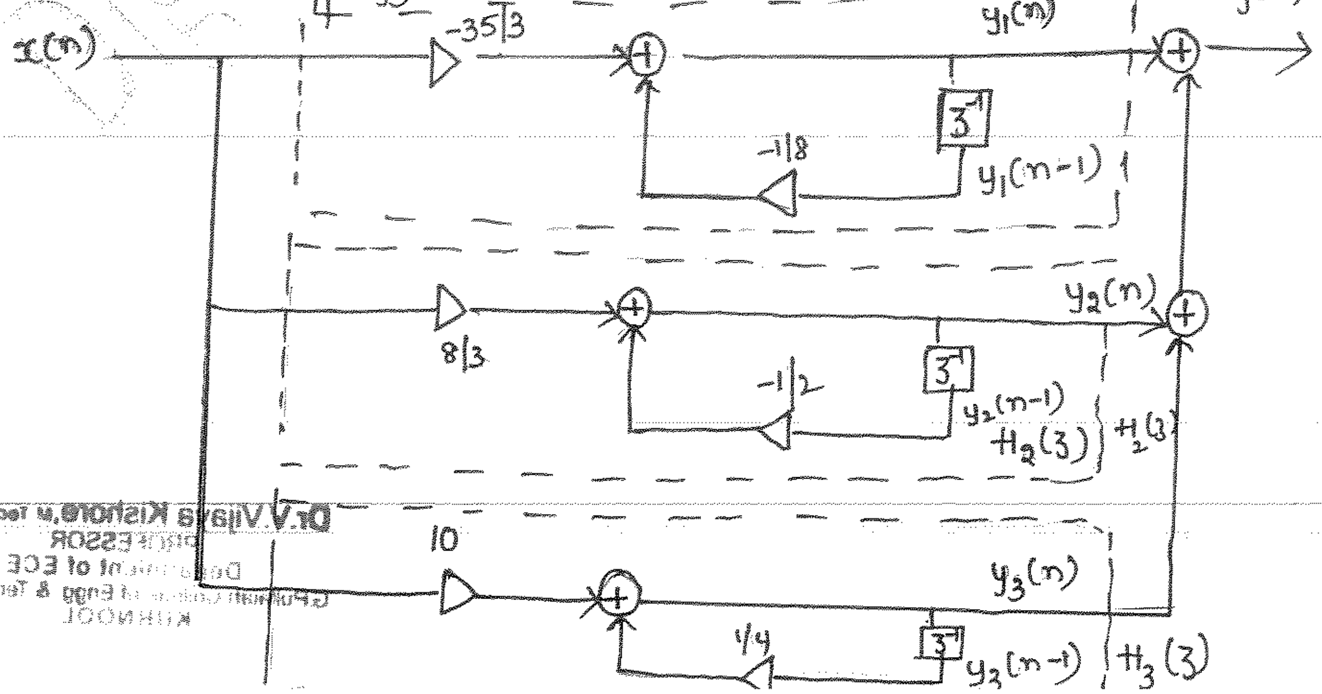
$$10X(z) = \left(1-\frac{1}{4}z^{-1}\right)Y_3(z)$$

Apply I.Z.T & shifting property

$$10x(n) = y_3(n) - \frac{1}{4}y_3(n-1)$$

$$y_3(n) = 10x(n) + \frac{1}{4}y_3(n-1)$$





Signal flowgraph :- Signal flowgraph is a graphical representation of relationship b/n the variables in a set of linear difference equation.

Basic elements of signal flowgraph are branches and nodes. Signal flow graph is a set of directed branches that connect at nodes.

Node is a system variable, which is equal to sum of incoming signals from branches connecting to the node.

Source Node are nodes that have no entering branches.

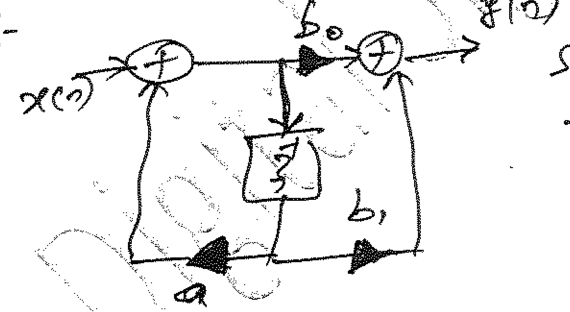
Sink Node that have only entering branches.

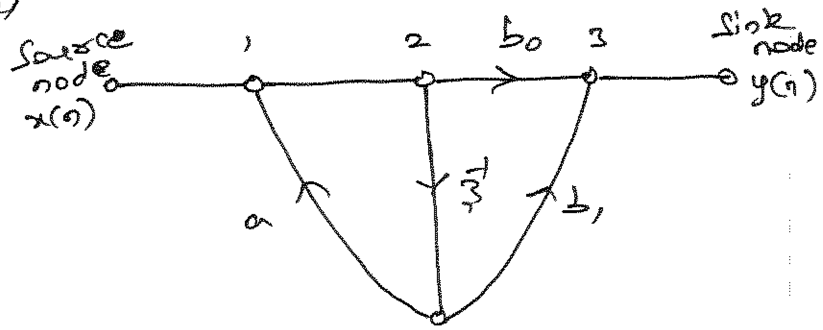
Signal out of a branch is equal to branch gain times the signal into the branch.

Arrow head shows the direction of the branch and branch gain is indicated on the arrow.

Delay is indicated by branch transmittance z^{-1}.

Eg :-





Transposition theorem and transposed Structure

Transpose of a structure is defined by

i) Reverse the direction of all branches in signal flow graph.

ii) Interchange the i/p's and o/p's

iii) Reverse the roles of all nodes in flowgraph

iv) Summing points become branching points

v) Branching points become summing points.

The system transfer function remain unchanged by transposition.

Eg: Determine the direct form II and Transposed direct form II for the system $y(n) = \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) + x(n) + x(n-1)$

Sol:- Given $y(n) = \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) + x(n) + x(n-1)$

Applying Z-T on both sides.

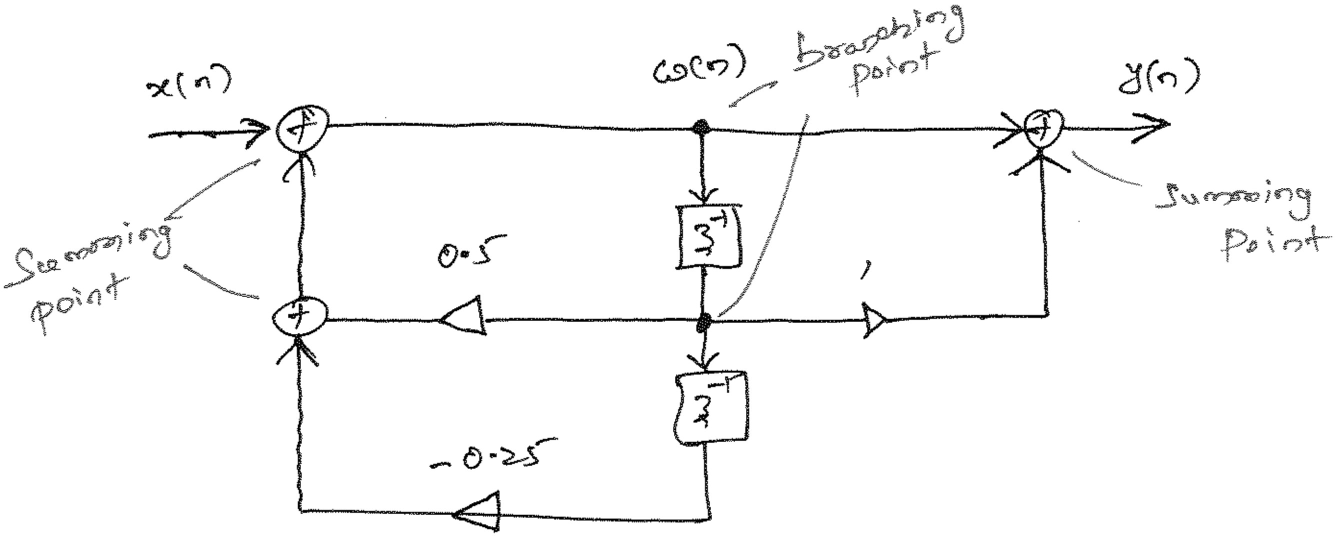
$$Y(z) = \frac{1}{2}z^{-1}Y(z) - \frac{1}{4}z^{-2}Y(z) + X(z) + z^{-1}X(z)$$

$$H(z) = \frac{1+z^{-1}}{1-0.5z^{-1}+0.25z^{-2}}$$

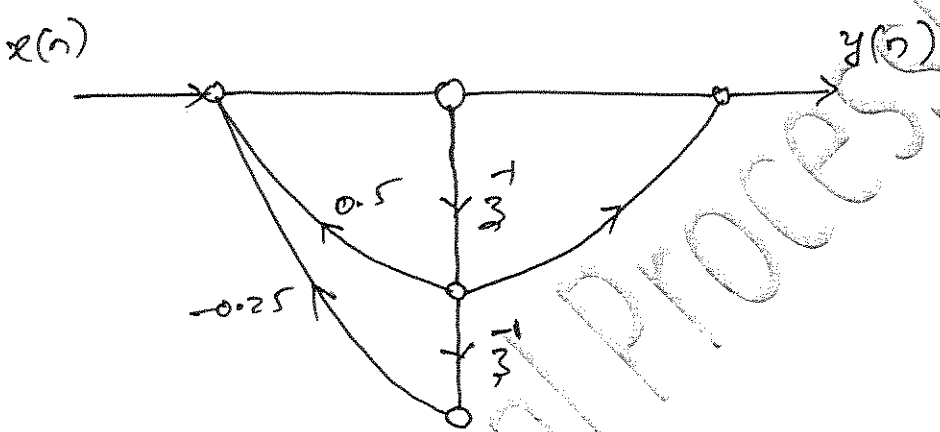
Direct form - II : $H(z) = \frac{W(z)}{X(z)} \times \frac{Y(z)}{W(z)}$

$$\frac{W(z)}{X(z)} = \frac{1}{1-0.5z^{-1}+0.25z^{-2}} \quad ; \quad \frac{Y(z)}{W(z)} = 1+z^{-1}$$

$w(n) = 0.5w(n-1) - 0.25w(n-2) + x(n)$ / $y(n) = w(n) + w(n-1)$

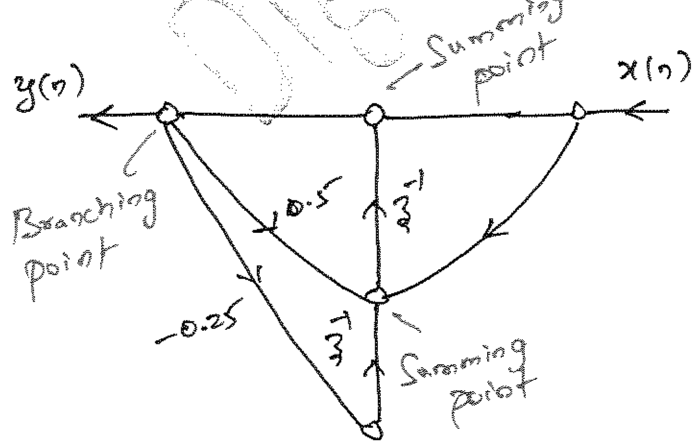


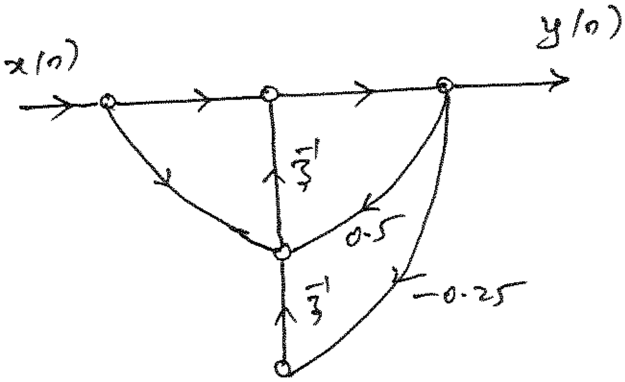
Signal flow graph is shown below



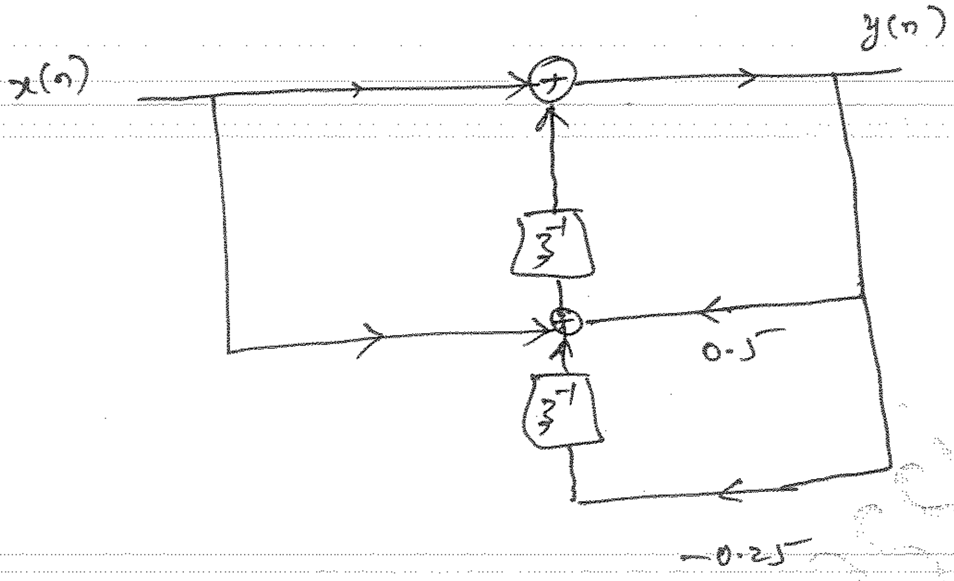
To get transposed direct form - II the operations are

(i) Change the direction of all branches
(ii) Interchange the input and output
(iii) Change the summing points to branching point & viceversa.





Transposition structure is shown below



FIR Realization

In General an N^{th} order FIR system is given by

$$y(n) = \sum_{m=0}^{N-1} b_m\, x(n-m)$$

$$= b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + \dots + b_{N-1}\, x(n-(N-1))$$

Z- domain representation of FIR system is

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}$$

Another form of transfer function is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n} = \frac{Y(z)}{X(z)}$$

interms of impulse response sequence.

There are $N-1$ zeros in the FIR system.

The various structures for realizing FIR system that gives direct relation between time domain and Z-domain equations are

1. Direct form / Transversal form
2. Cascade realization
3. Linear phase realization

1. Direct form / Transversal Structure

FIR system is given by (time domain)

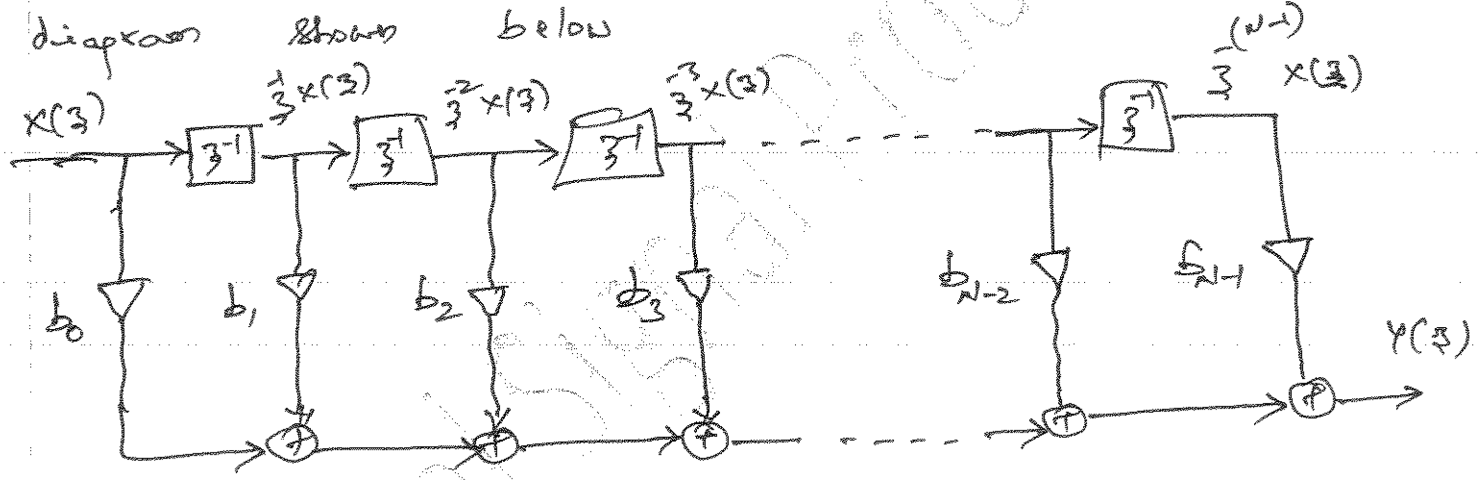
$$y(n) = \sum_{m=0}^{N-1} b_m\, x(n-m)$$

$$= b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + \ldots + b_{N-1}\, x(n-(N-1)) \quad \text{— (1)}$$

taking Z-transform we get Z-domain representation

$$Y(z) = b_0\, X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \ldots + b_{N-1} z^{-(N-1)} X(z) \quad \text{— (2)}$$

Equation (1) and (2) can be directly represented by block diagram shown below



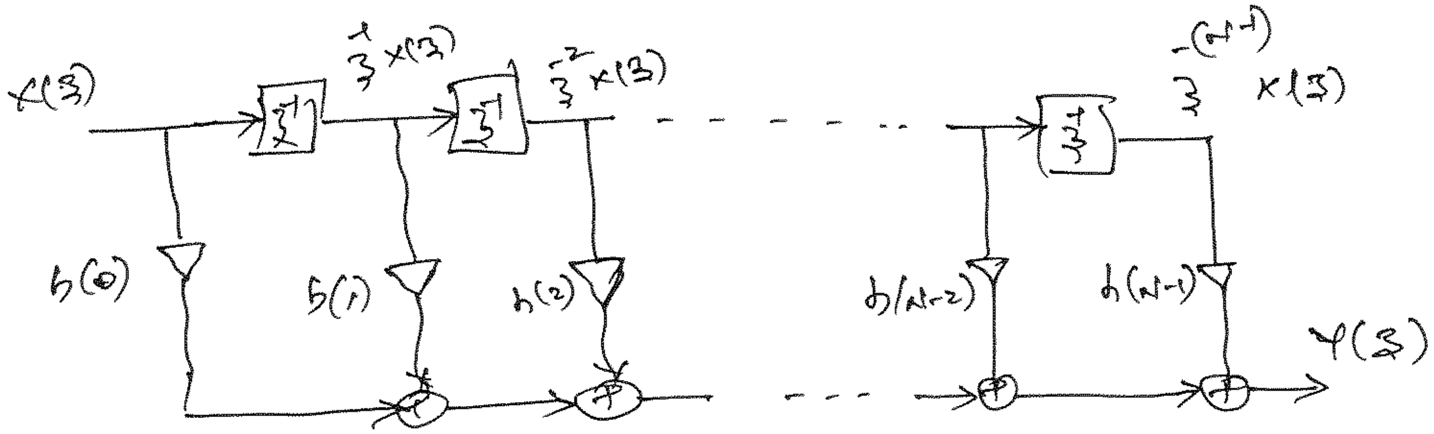
Equation (2) can also be expressed in impulse response $h(n)$ as $H(z) = \sum_{m=0}^{N-1} h(n) \cdot z^{-n}$

$$H(z) = \frac{Y(z)}{X(z)} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \ldots + h(N-1) z^{-(N-1)}$$

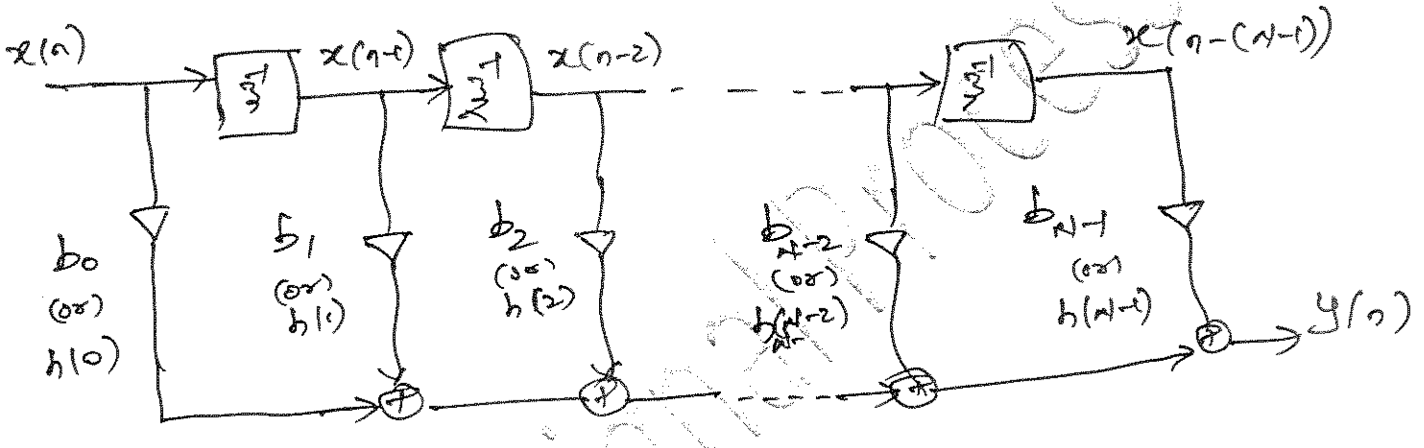
$$Y(z) = X(z) \left[h(0) + h(1) z^{-1} + h(2) z^{-2} + \ldots + h(N-1) z^{-(N-1)} \right] \quad \text{— (3)}$$

Applying inverse Z.T and shifting property.

$$y(n) = h(0)\, x(n) + h(1)\, x(n-1) + h(2)\, x(n-2) + \ldots + h(N-1)\, x(n-(N-1)) \quad \text{— (4)}$$



Similarly structures for time domain representation Eq ① and Eq ④ {Eq ① – factors are $b_0, b_1, \dots b_{N-1}$
Eq ④ – factors are $h(0), h(1), h(2) \dots h(N-1)$.}



No. of multiplications = N

No. of additions = N-1

No. of delays = N-1

No. of Memory locations to store delayed signals = N-1

2. Cascade form Realization of FIR system

Transfer function of FIR systems is

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}$$

Transfer function of FIR system is $(N-1)^{th}$ polynomial in z. This polynomial can be factorized into 1^{st} and 2^{nd} order and transfer function is expressed as

$$H(z) = \frac{Y(z)}{X(z)} = H_1(z) \times H_2(z) \times H_3(z) \times \cdots \times H_m(z)$$

$$= \prod_{i=1}^{m} H_i(z)$$

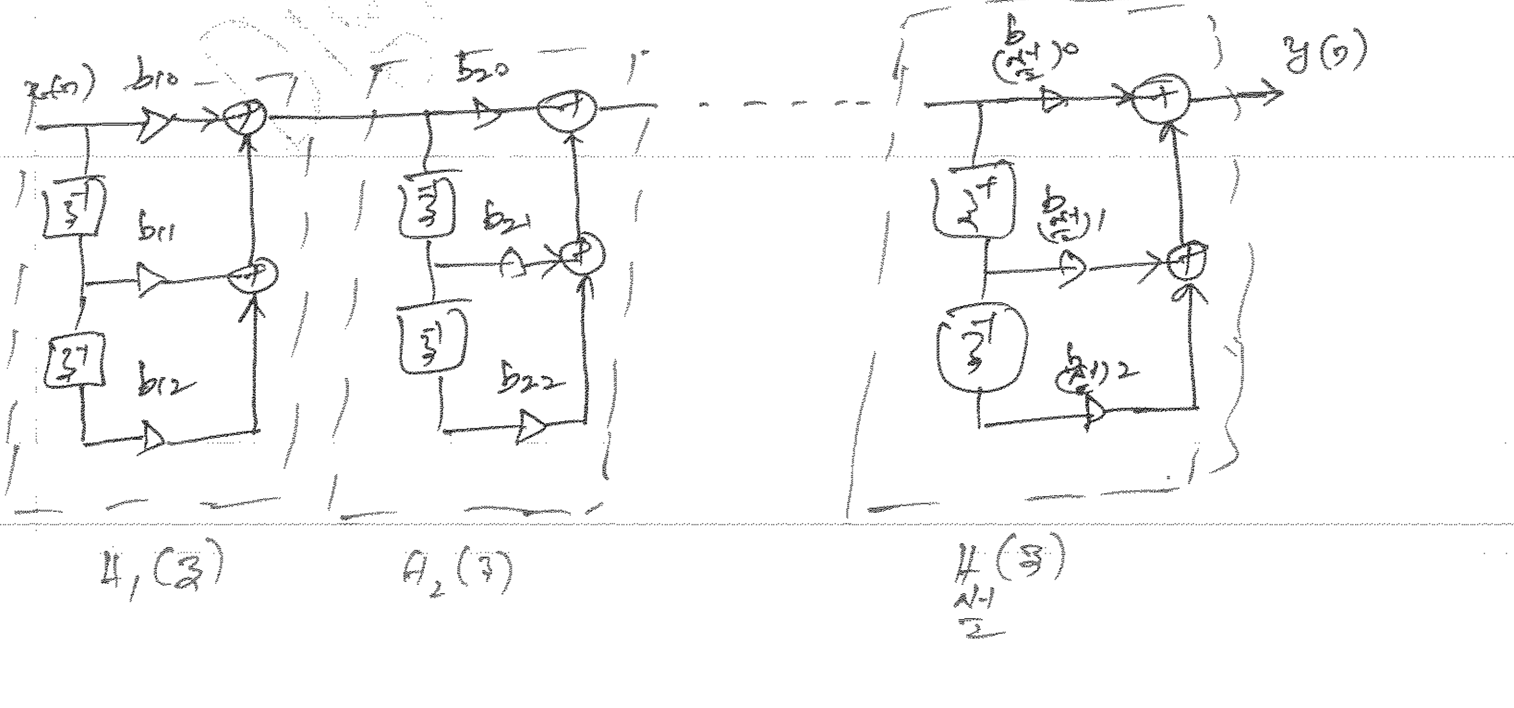
$H_i(z)$ can be either 1^{st} order & 2^{nd} order.

Individual $H_i(z)$ can be realized either in direct form structure or linear phase structure.

Case i: N - odd :- N is odd, $(N-1)$ is even, $H_i(z)$ are second order transfer function sections. There are $\left(\frac{N-1}{2}\right)$ number of sections.

$$H(z) = \prod_{k=1}^{\left(\frac{N-1}{2}\right)} \left(b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}\right)$$

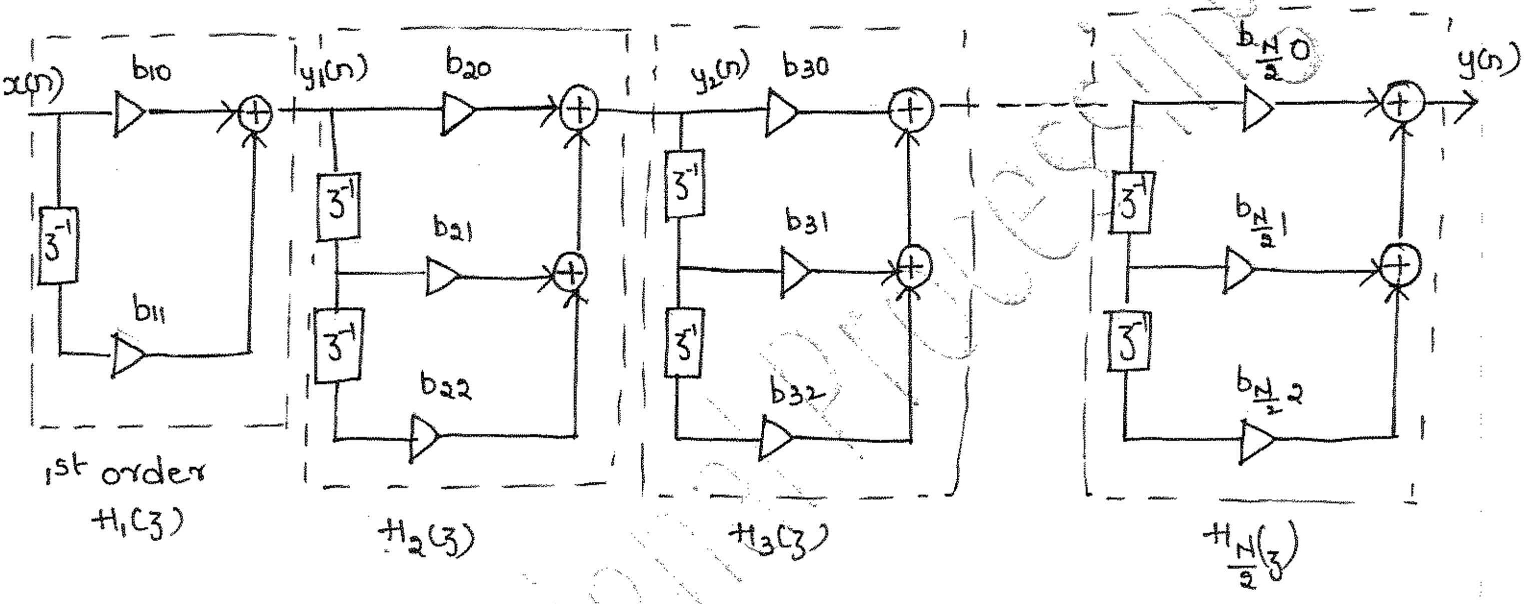
$$H(z) = \left(b_{10} + b_{11} z^{-1} + b_{12} z^{-2}\right)\left(b_{20} + b_{21} z^{-1} + b_{22} z^{-2}\right) \cdots \left(b_{\left(\frac{N-1}{2}\right)0} + b_{\left(\frac{N-1}{2}\right)1} z^{-1} + b_{\left(\frac{N-1}{2}\right)2} z^{-2}\right)$$



Case ii : N - odd : There are $\frac{N}{2}$ factors which are second order and one first order transfer function section. Each section is to be realized in direct form (or) linear phase form

$$H(z) = \left(b_{10} + b_{11} z^{-1}\right) \prod_{k=2}^{\frac{N}{2}} \left(b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}\right)$$

$$H(z) = \left(b_{10} + b_{11} z^{-1}\right)\left(b_{20} + b_{21} z^{-1} + b_{22} z^{-2}\right)\left(b_{30} + b_{31} z^{-1} + b_{32} z^{-2}\right) \text{-----} \left(b_{\left(\frac{N}{2}\right)0} + b_{\left(\frac{N}{2}\right)1} z^{-1} + b_{\left(\frac{N}{2}\right)2} z^{-2}\right)$$



NOTE: In time domain replace $X(z)$ by $x(n)$ & $Y(z)$ by $y(n)$

3. Linear phase realization :

Consider impulse response $h(n) = \{b_0, b_1, b_2 \text{ -- } b_{N-1}\}$

In FIR system linear phase response with symmetrical impulse response. i.e $\boxed{h(n) = h(N-1-n)}$

When impulse response is symmetric, the samples of impulse response will satisfy $b_n = b_{N-1-n}$

By using above condition, the number of multipliers required for realization can be reduced. Hence linear phase realization is also called realization with minimum number of multipliers.

Consider FIR transfer function

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}$$

Case i : N - even :-

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}$$

$$= \sum_{m=0}^{N-1} b_m \cdot z^{-m} = \sum_{m=0}^{\frac{N}{2}-1} b_m z^{-m} + \sum_{m=\frac{N}{2}}^{N-1} b_m z^{-m}$$

let $P = N-1-m \Rightarrow m = N-1-P$

$m = \frac{N}{2} \Rightarrow P = N-1-\frac{N}{2} = \frac{N}{2}-1$

$m = N-1 \Rightarrow P = N-1-(N-1) = 0$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \sum_{m=0}^{\frac{N}{2}-1} b_m z^{-m} + \sum_{P=0}^{\frac{N}{2}-1} b_{N-1-P} \, z^{-(N-1-P)}$$

$$= \sum_{m=0}^{\frac{N}{2}-1} b_m z^{-m} + \sum_{m=0}^{\frac{N}{2}-1} b_{N-1-m} \, z^{-(N-1-m)}$$

$$= \sum_{m=0}^{\frac{N}{2}-1} b_m \cdot z^{-m} + \sum_{m=0}^{\frac{N}{2}-1} b_m \cdot z^{-(N-1-m)}$$

$(\because b_m = b_{N-1-m}$ due to symmetry$)$

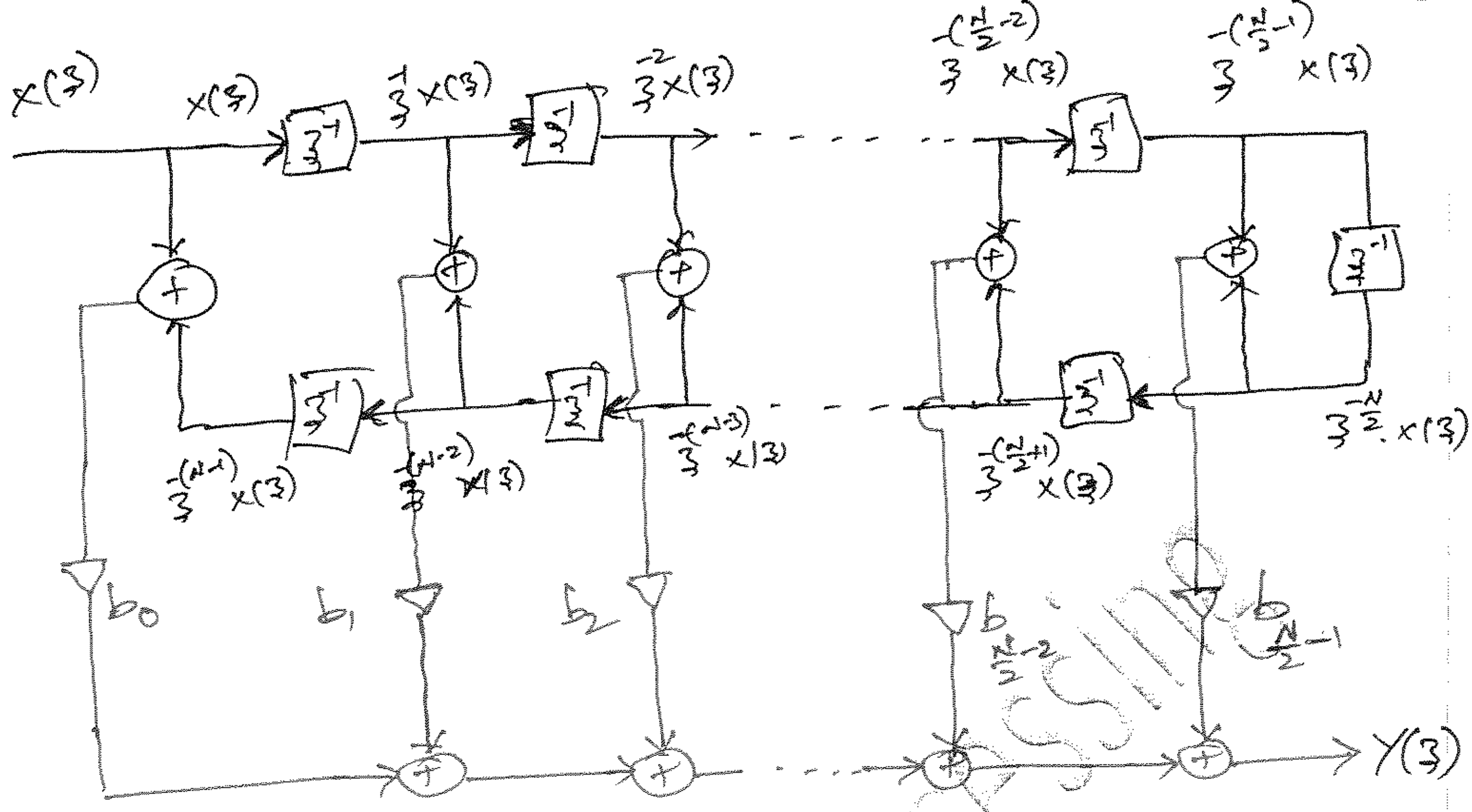
$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \sum_{m=0}^{\frac{N}{2}-1} b_m \left(z^{-m} + z^{-(N-1-m)} \right)}$$

(or)

Similarly $\Rightarrow$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \sum_{m=0}^{\frac{N}{2}-1} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]}$$

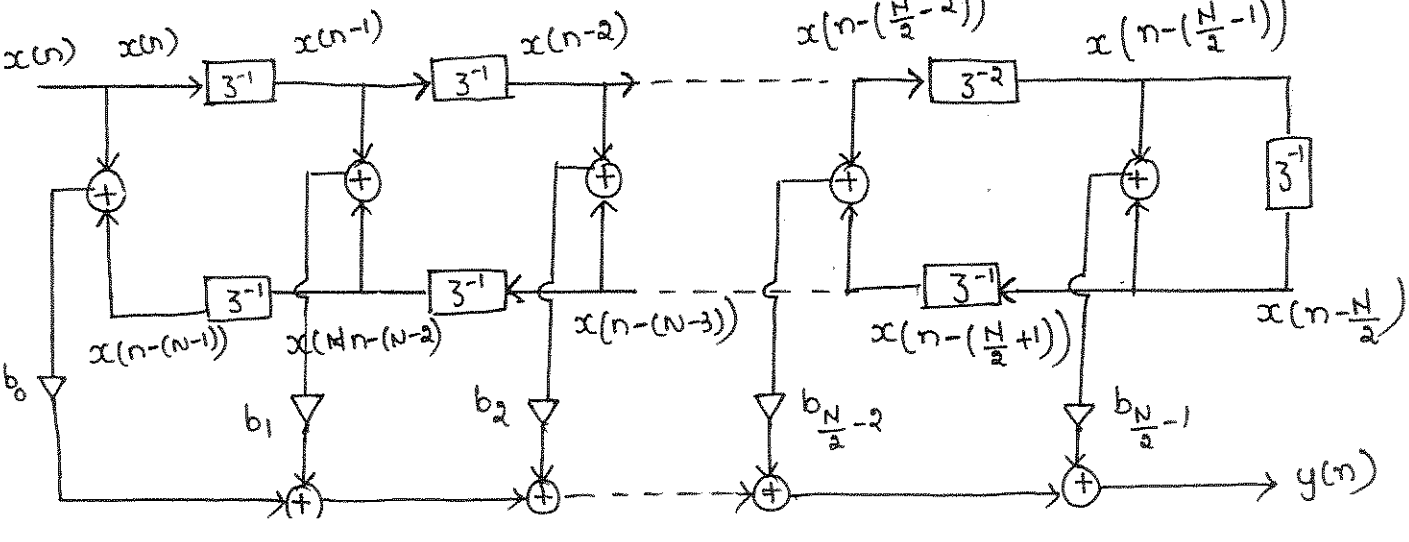
if $H(z) = \sum_{n=0}^{N-1} h(n) \cdot z^{-n}$ is taken



$$Y(z) = b_0\left(X(z) + z^{-(N-1)}X(z)\right) + b_1\left(z^{-1}X(z) + z^{-(N-2)}X(z)\right) + \ldots$$

$$\ldots + b_{\frac{N}{2}-2}\left(z^{-(\frac{N}{2}-2)}\cdot X(z) + z^{-(\frac{N}{2}+1)}X(z)\right) + b_{\frac{N}{2}-1}\left(z^{-(\frac{N}{2}-1)}X(z) + z^{-\frac{N}{2}}X(z)\right)$$

For the above realization of N^{th} order FIR DTS, for even values of 'N', the number of multiplications is '$\frac{N}{2}$' and number of additions is 'N-1'.
Number of delay is (N-1) and so memory locations required is also (N-1) to store delayed signals.



Case ii: N-odd :-

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)} = \sum_{m=0}^{N-1} b_m \cdot z^{-m}$$

$$= \sum_{m=0}^{\frac{N-3}{2}} b_m \cdot z^{-m} + b_{\frac{N-1}{2}} \cdot z^{-\left(\frac{N-1}{2}\right)} + \sum_{m=\frac{N+1}{2}}^{N-1} b_m \cdot z^{-m}$$

let $p = N-1-m \Rightarrow m = N-1-p$

$m = \frac{N+1}{2} \Rightarrow p = N-1-\frac{N+1}{2} = \frac{N-3}{2}$

$m = N-1 \Rightarrow p = N-1-(N-1) = 0$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \sum_{m=0}^{\frac{N-3}{2}} b_m \cdot z^{-m} + b_{\frac{N-1}{2}} \cdot z^{-\left(\frac{N-1}{2}\right)} + \sum_{p=0}^{\frac{N-3}{2}} b_{N-1-p} \cdot z^{-(N-1-p)}$$

$$= \sum_{m=0}^{\frac{N-3}{2}} b_m \cdot z^{-m} + b_{\frac{N-1}{2}} \cdot z^{-\left(\frac{N-1}{2}\right)} + \sum_{m=0}^{\frac{N-3}{2}} b_{N-1-m} \cdot z^{-(N-1-m)}$$

$$= \sum_{m=0}^{\frac{N-3}{2}} b_m \cdot z^{-m} + b_{\frac{N-1}{2}} \cdot z^{-\left(\frac{N-1}{2}\right)} + \sum_{m=0}^{\frac{N-3}{2}} b_m \cdot z^{-(N-1-m)} \qquad (p = m)$$

($b_m = b_{N-1-m}$, Symmetry

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = b_{\frac{N-1}{2}} \cdot z^{-\left(\frac{N-1}{2}\right)} + \sum_{m=0}^{\frac{N-3}{2}} b_m \left(z^{-m} + z^{-(N-1-m)} \right)}$$

$$Y(z) = X(z)\left[b_{\frac{N-1}{2}} \cdot z^{-\left(\frac{N-1}{2}\right)} \right] + b_0 \left(X(z) + z^{-(N-1)} X(z) \right) + b_1 \left(z^{-1} X(z) + z^{-(N-2)} X(z) \right) +$$

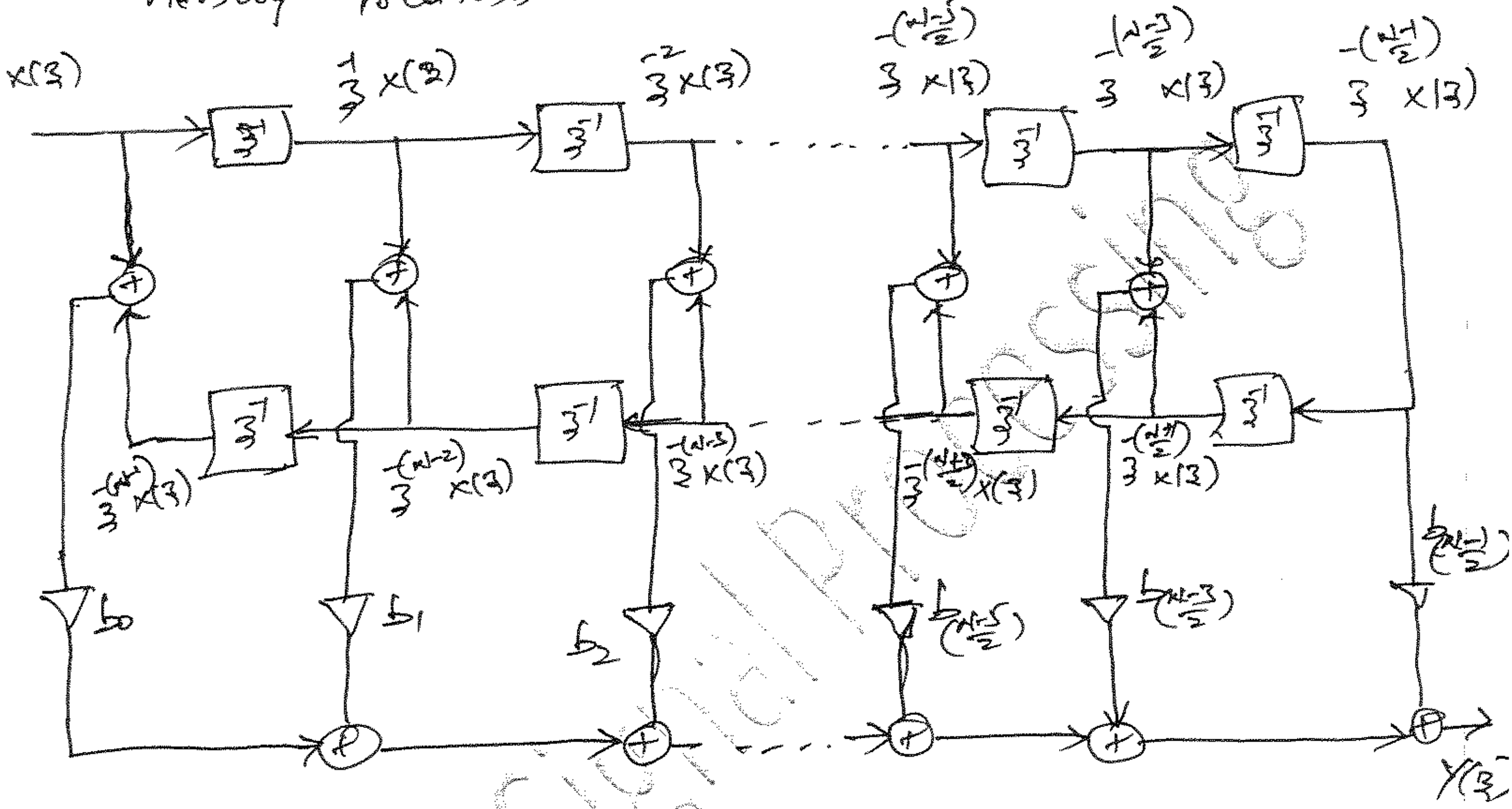
$$\dots + b_{\frac{N-5}{2}} \left(z^{-\left(\frac{N-5}{2}\right)} X(z) + z^{-\left(\frac{N+3}{2}\right)} \cdot X(z) \right) + b_{\frac{N-3}{2}} \left(z^{-\left(\frac{N-3}{2}\right)} X(z) + z^{-\left(\frac{N+1}{2}\right)} X(z) \right)$$

For N-odd and N^{th} order FIR System realized below the number of multiplications are $\left(\frac{N+1}{2}\right)$

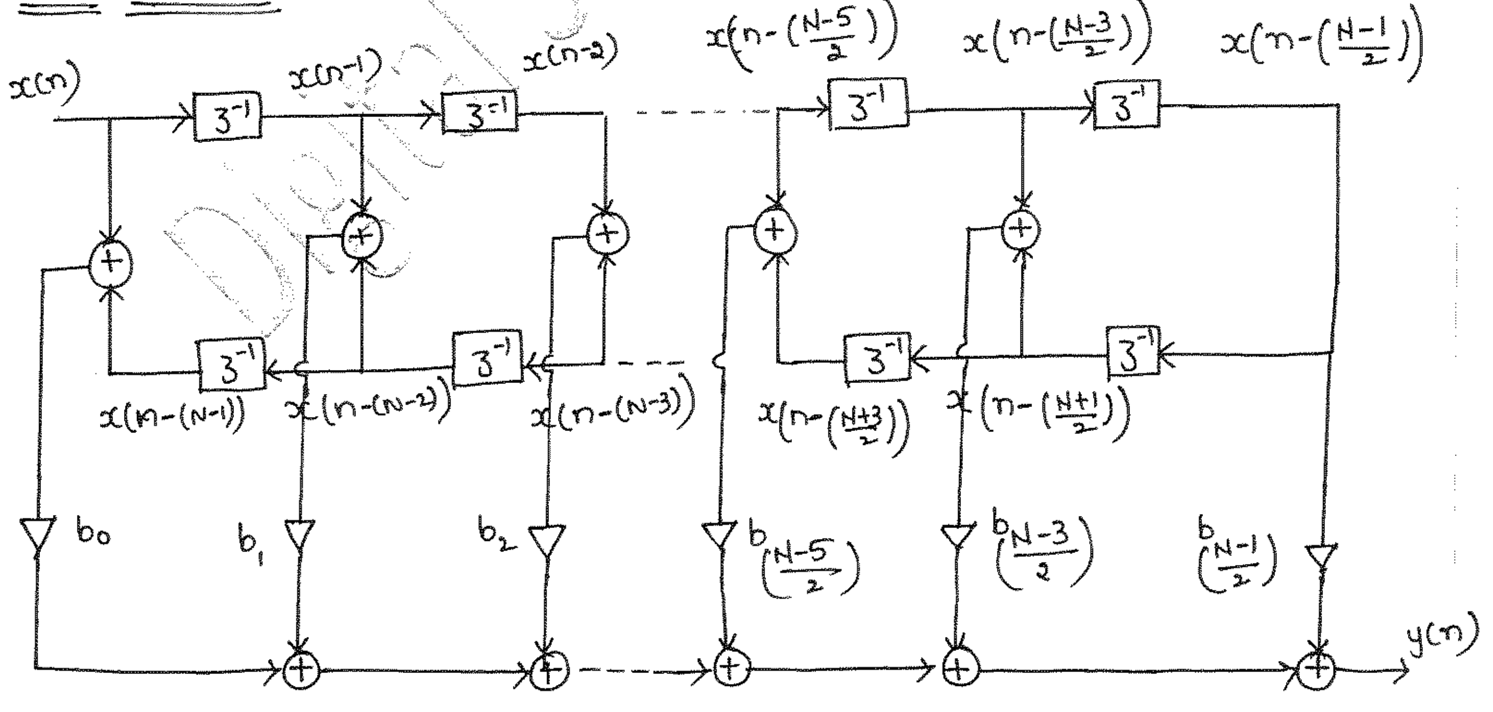
number of additions are $(N-1)$

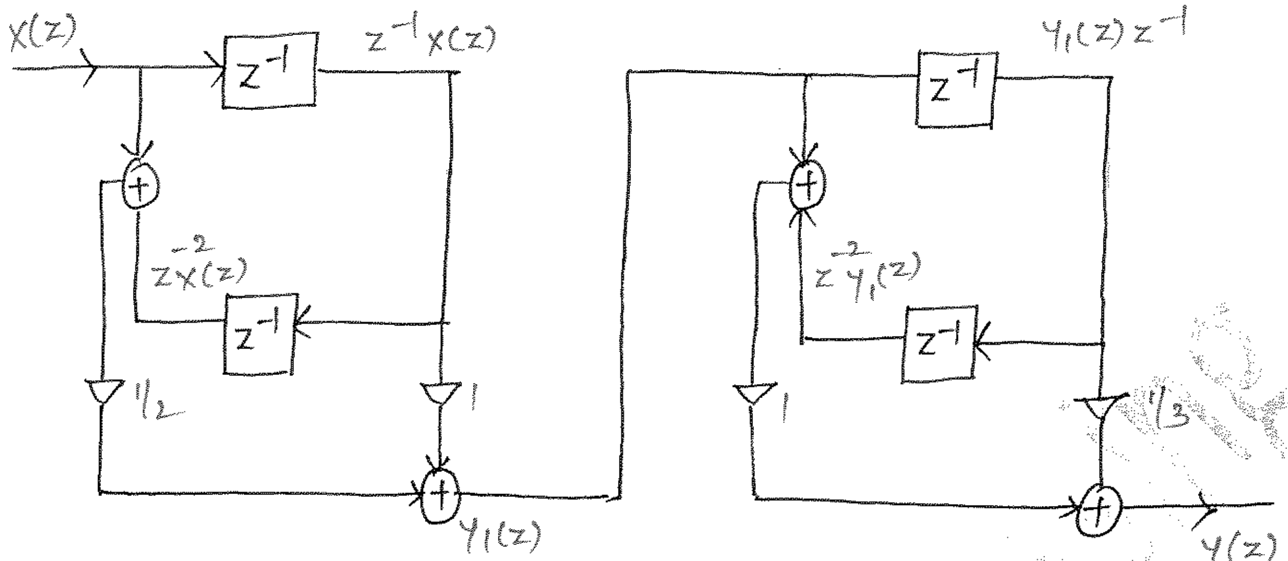
number of delays are $(N-1)$

memory locations is also $(N-1)$ to store digital signals



TIME DOMAIN:





4. Obtain the Cascade realisation of system using Direct form and Linear phase

$H(z) = (1+2z^{-1}-z^{-2})\,(1+z^{-1}-z^{-2})$

Sol: Given $H(z) = (1+2z^{-1}-z^{-2})\,(1+z^{-1}-z^{-2}) \longrightarrow$ eqn(1)

from above eqn $\frac{N-1}{2} = 2$

$N-1 = 4 \Rightarrow N = 5$ (Odd).

Transfer function $H(z)$ for odd N is given as

$$H(z) = \sum_{K=1}^{\left(\frac{N-1}{2}\right)} (b_{K0} + b_{K1}z^{-1} + b_{K2}z^{-2})$$

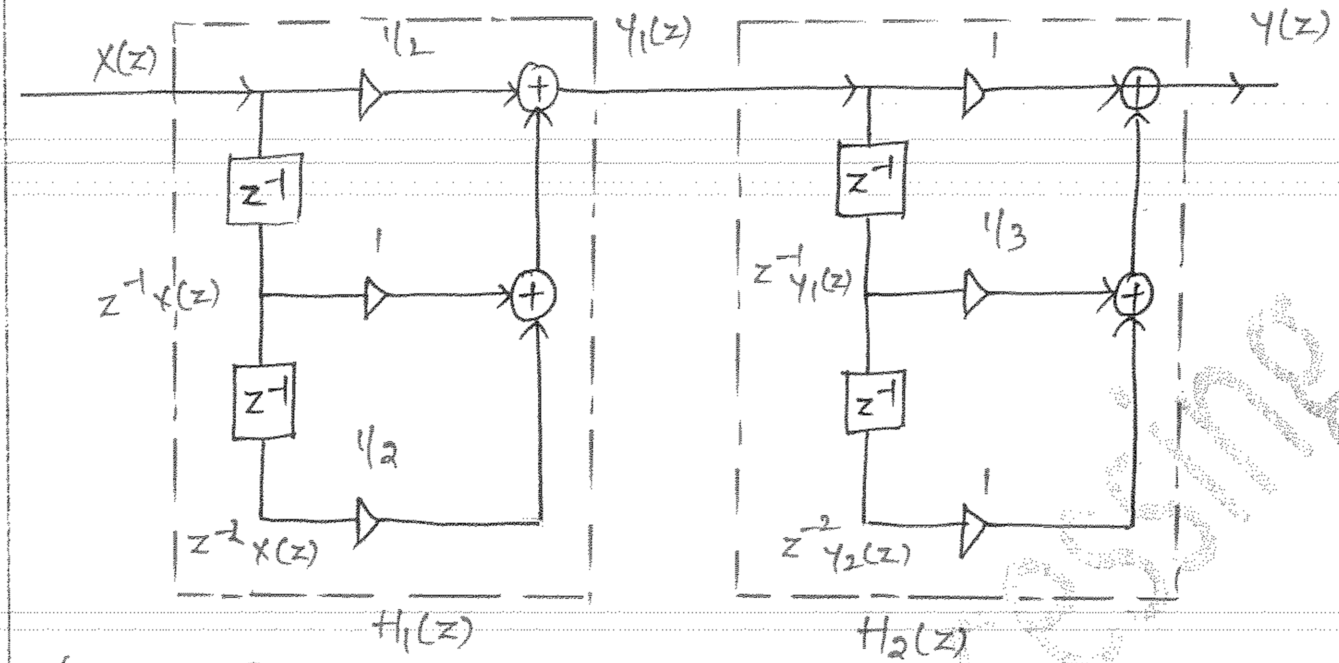
$$= \sum_{K=1}^{2} (b_{K0} + b_{K1}z^{-1} + b_{K2}z^{-2})$$

$H(z) = (b_{10}+b_{11}z^{-1}+b_{12}z^{-2})\,(b_{20}+b_{21}z^{-1}+b_{22}z^{-2}) \longrightarrow$ eqn(2)

Comparing eqn (1) & (2)

$b_{10}=1,\ b_{11}=2,\ b_{12}=-1$

$b_{20}=1,\ b_{21}=1,\ b_{22}=-1$



Linear Phase Realization:

$$H(z) = \left(\frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}\right)\left(1 + \frac{1}{3} z^{-1} + z^{-2}\right)$$

$$H_1(z) = \frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2} \rightarrow (1) \; ; \quad N-1 = 2$$

$$N = 3.$$

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} \rightarrow (2) \; ; \text{ for } N = 3$$

Compare (1) & (2) $\quad b_0 = 1/2, \; b_1 = 1, \; b_2 = 1/2$

$$b_0 = b_2 = 1/2, \; b_1 = 1$$

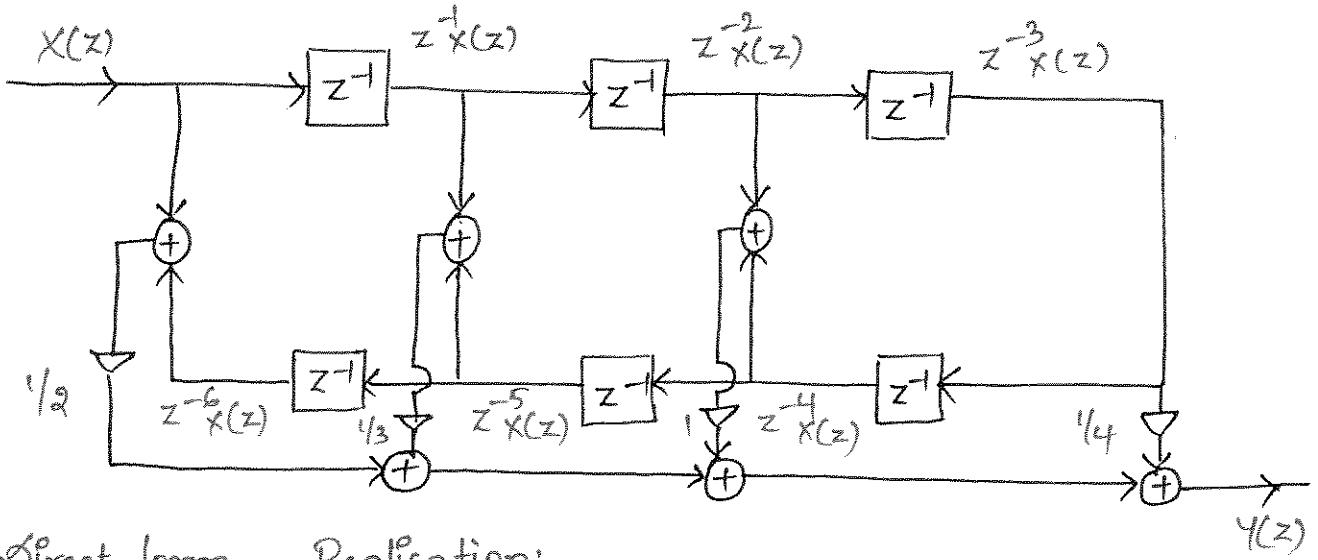
$$H_1(z) = \frac{1}{2}(1 + z^{-2}) + z^{-1} = \frac{Y_1(z)}{X(z)}$$

$$Y_1(z) = \frac{1}{2}\left(X(z) + X(z) z^{-2}\right) + X(z) z^{-2}$$

||ly $\quad H_2(z) = \dfrac{Y(z)}{Y_1(z)} = 1 + 1/3 z^{-1} + z^{-2}.$

$$\frac{Y(z)}{Y_1(z)} = (1 + z^{-2}) + \frac{1}{3} z^{-1}$$

$$Y(z) = \left(Y_1(z) + Y_1(z) z^{-2}\right) + \frac{1}{3} Y_1(z) z^{-1}$$



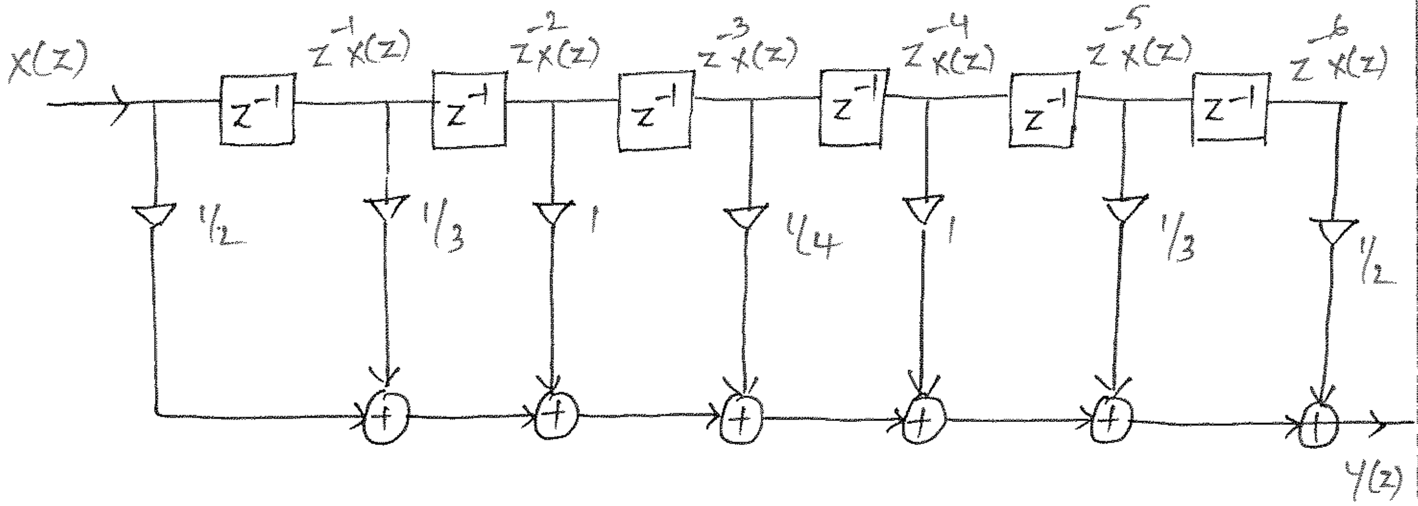
Direct form Realisation:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$$

$$Y(z) = \frac{1}{2}X(z) + \frac{1}{3}X(z)z^{-1} + X(z)z^{-2} + \frac{1}{4}X(z)z^{-3} + X(z)z^{-4} + \frac{1}{3}X(z)z^{-5} + \frac{1}{2}X(z)z^{-6}.$$

Apply I.Z.T & by using shifting property

$$y(n) = \frac{1}{2}x(n) + \frac{1}{3}x(n-1) + x(n-2) + \frac{1}{4}x(n-3) + x(n-4) + \frac{1}{3}x(n-5) + \frac{1}{2}x(n-6).$$



Replace $X(z) = x(n)$, $Y(z) = y(n)$ in Time Domain.

Dr.V.Vijaya Kishore, M.Tech., Ph.D.
PROFESSOR
Department of ECE
G.Pullaiah College of Engg. & Technology
KURNOOL.

3. Obtain the cascade Realisation with Direct form and Linear phase.

$H(z) = \left(\frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}\right)\left(1 + \frac{1}{3} z^{-1} + z^{-2}\right)$

Sol: Given $H(z) = \left(\frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}\right)\left(1 + \frac{1}{3} z^{-1} + z^{-2}\right)$

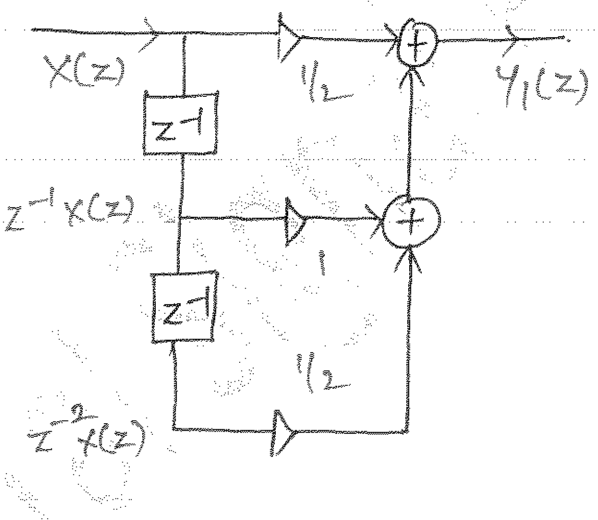
Cascade form $H(z) = \sum_{i} \prod_{i=1}^{M} H_i(z)$

$H(z) = H_1(z) \cdot H_2(z)$ Acc to the problem.

Direct form Realisation:

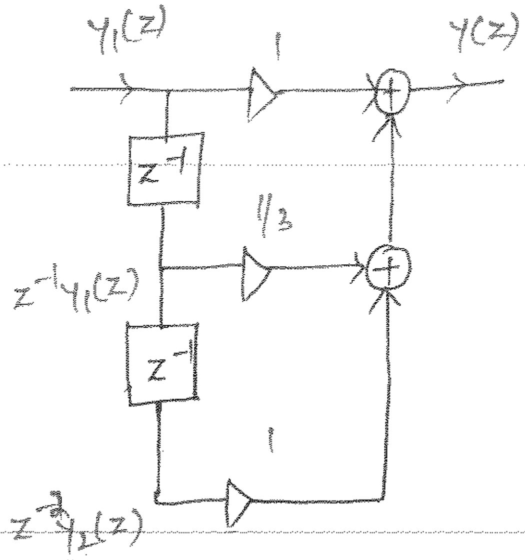
$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}$

$Y_1(z) = \frac{1}{2} X(z) + X(z) z^{-1} + \frac{1}{2} X(z) \cdot z^{-2}$



$H_2(z) = \frac{Y(z)}{Y_1(z)} = 1 + \frac{1}{3} z^{-1} + z^{-2}$

$Y(z) = Y_1(z) + \frac{Y_1(z) z^{-1}}{3} + Y_1(z) \cdot z^{-2}$



$\left\{ \because \text{Replace } X(z) = x(n),\ Y_1(z) = y_1(n),\ Y(z) = y(n) \text{ in Time domain} \right\}$

2. Realise the system function in Linear phase & Direct form.

$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{3}z^{-5} + z^{-4} + \frac{1}{2}z^{-6}.$$

Sol: Linear phase realisation:

Given

$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6} \longrightarrow (1)$$

W.K.T $H(z) = \sum_{m=0}^{N-1} b_m z^{-m} = \sum_{n=0}^{6} b_m z^{-m}$

Here $N-1 = 6$

$N = 7$ (odd)

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} \longrightarrow \text{eqn}(2)$$

Comparing eqn(1) & (2)

$b_0 = 1/2,\ b_1 = 1/3,\ b_2 = 1,\ b_3 = 1/4,\ b_4 = 1,\ b_5 = 1/3,\ b_6 = 1/2$

$$H(z) = \frac{Y(z)}{X(z)} = b_0(1+z^{-6}) + b_1(z^{-1}+z^{-5}) + b_2(z^{-2}+z^{-4}) + b_3 z^{-3}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{2}(1+z^{-6}) + \frac{1}{3}(z^{-1}+z^{-5}) + 1(z^{-2}+z^{-4}) + \frac{1}{4}z^{-3}$$

$$Y(z) = \frac{1}{2}\left[X(z) + X(z)z^{-6}\right] + \frac{1}{3}\left[X(z)z^{-1} + X(z)z^{-5}\right] + \left[X(z)z^{-2} + X(z)z^{-4}\right] + \frac{X(z)}{4}z^{-3}.$$

1. Determine direct form realisation of FIR system
$H(z) = 1+2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

Given Transfer function

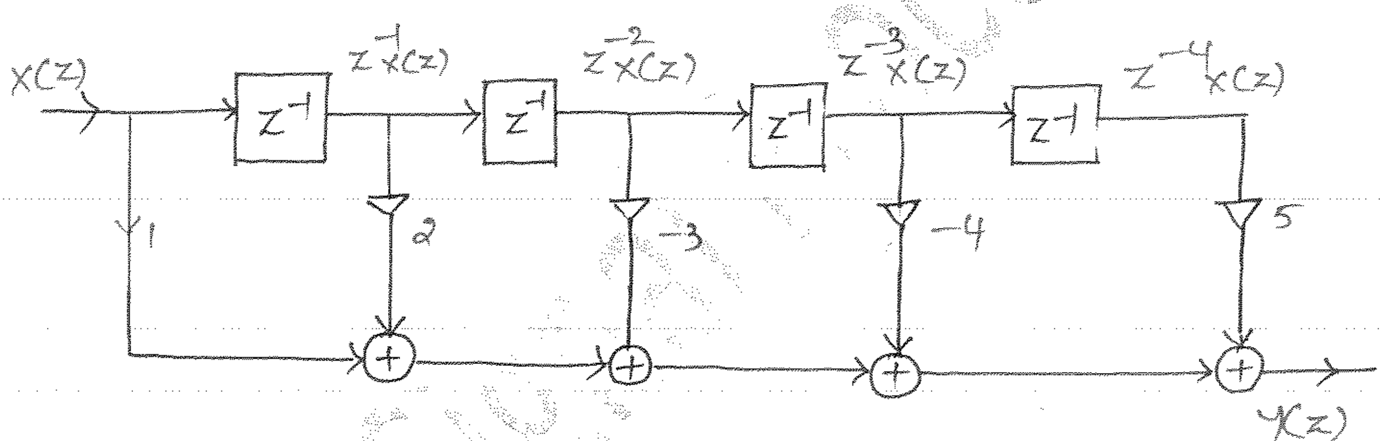
$$H(z) = 1+2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$

$$H(z) = \frac{Y(z)}{X(z)} = 1+2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$

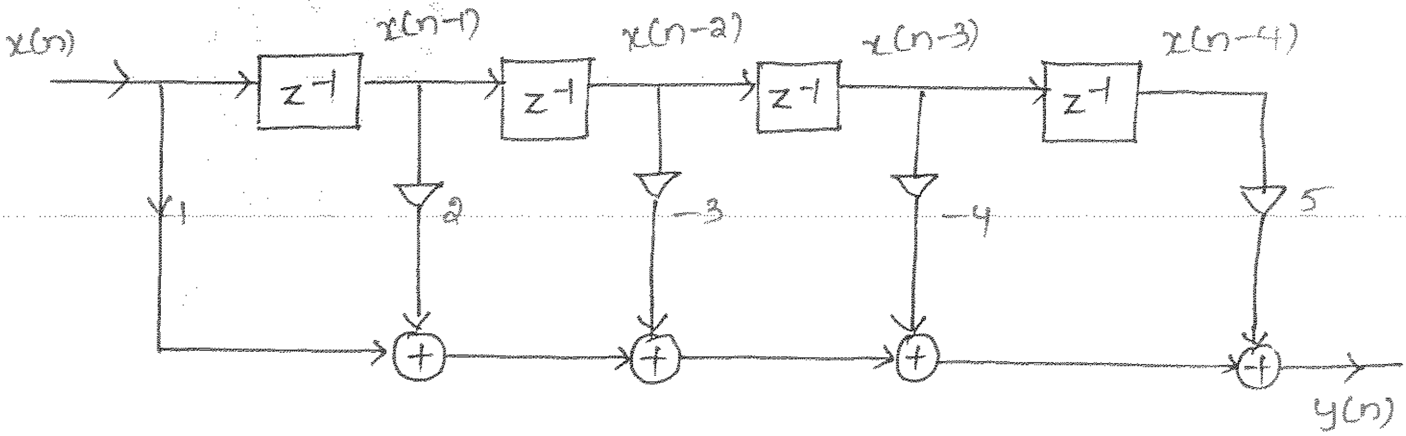
$$Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z) - 4z^{-3}X(z) + 5z^{-4}X(z)$$

Applying Inverse Z-Transform and using shifting property

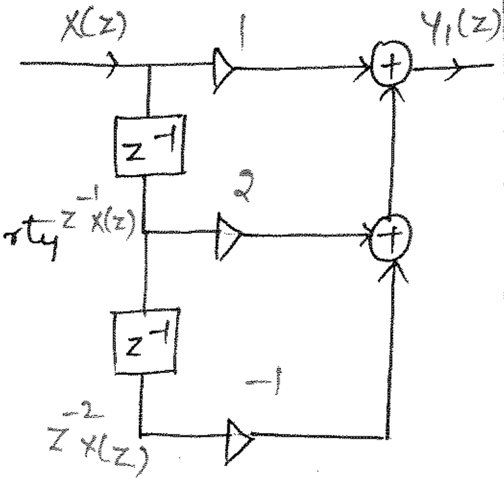
$$y(n) = x(n) + 2x(n-1) - 3x(n-2) - 4x(n-3) + 5x(n-4)$$



In time Domain:



$$H_1(z) = \frac{Y_1(z)}{X(z)} = 1 + 2z^{-1} - z^{-2}.$$



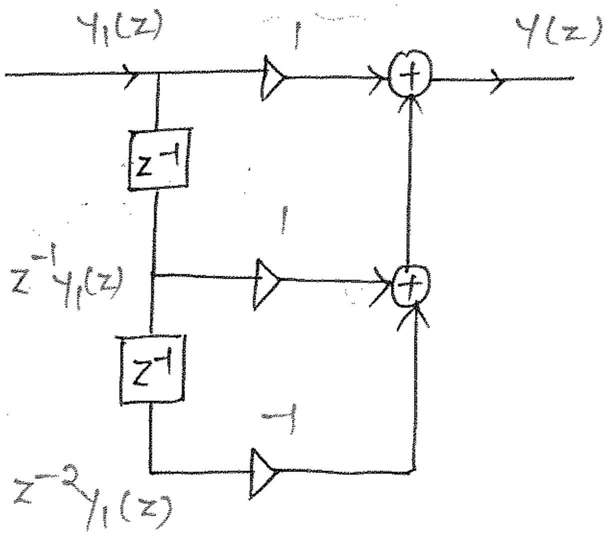
$$Y_1(z) = X(z) + 2X(z)z^{-1} - X(z)z^{-2}$$

Apply I.Z.T & using shifting property

$$y_1(n) = x(n) + 2x(n-1) - x(n-2)$$

$\parallel^{ly}$ $H_2(z) = \frac{Y(z)}{Y_1(z)}$

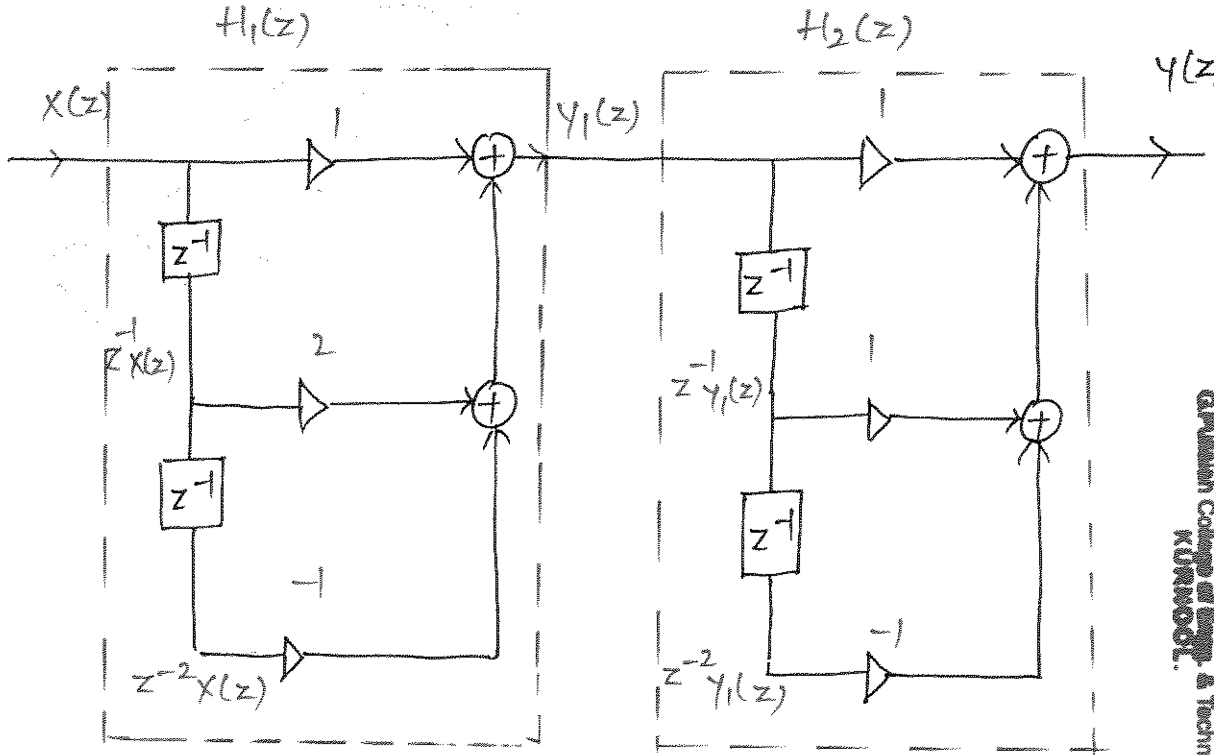
$$\frac{Y(z)}{Y_1(z)} = 1 + z^{-1} - z^{-2}$$



$$Y(z) = Y_1(z) + Y_1(z)z^{-1} - Y_1(z)z^{-2}.$$

Apply I.Z.T & Using shifting property

$$y(n) = y_1(n) + y_1(n-1) - y_1(n-2)$$



Dr.V.Vijaya Kishore, M.Tech.,Ph.D.
PROFESSOR
Department of ECE
G.Pullaiah College of Engg. & Technology
KURNOOL.