

UNIT-II

MAGNETOSTATICS

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- Biot - Savart's Law
- Ampere's Circuital Law and Applications
- Magnetic Flux Density
- Maxwell's Equations for Magnetostatic Fields
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Maxwell's Equations (Time Varying Fields):

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- Transformer EMF
- Displacement Current Density
- Maxwell's Equations in Different Final Forms
- Conditions at a Boundary Surface: Dielectric - Dielectric,
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Introduction:

In previous chapters we have seen that an electrostatic field is produced by static or stationary charges. The relationship of the steady magnetic field to its sources is much more complicated.

The source of steady magnetic field may be a permanent magnet, a direct current or an electric field changing with time. In this chapter we shall mainly consider the magnetic field produced by a direct current. The magnetic field produced due to time varying electric field will be discussed later.

There are two major laws governing the magneto static fields are:

- Biot-Savart Law
- Ampere's Law

Usually, the magnetic field intensity is represented by the vector \vec{H} . It is customary to represent the direction of the magnetic field intensity (or current) by a small circle with a dot or cross sign depending on whether the field (or current) is out of or into the page as shown in Fig. 2.1.

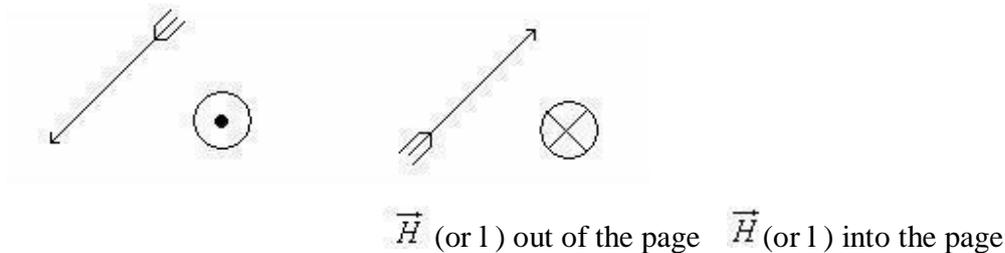
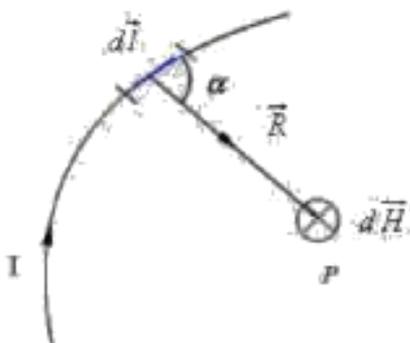


Fig. Representation of magnetic field (or current)

Biot- Savart's Law:

This law relates the magnetic field intensity dH produced at a point due to a differential current element $Id\vec{l}$ as shown in Fig.



The magnetic field intensity $d\vec{H}$ at P can be written as,

$$d\vec{H} = \frac{I d\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{I d\vec{l} \times \vec{R}}{4\pi R^3}$$

$$dH = \frac{I dl \sin \alpha}{4\pi R^2}$$

where $R = |\vec{R}|$ is the distance of the current element from the point P.

The value of the constant of proportionality 'K' depends upon a property called permeability of the medium around the conductor. Permeability is represented by symbol 'm' and the constant 'K' is expressed in terms of 'm' as

Thus

$$dB = \frac{\mu}{4\pi} \frac{I dl \sin \theta}{r^2}$$

Magnetic field 'B' is a vector and unless we give the direction of 'dB', its description is not complete. Its direction is found to be perpendicular to the plane of 'r' and 'dl'.

If we assign the direction of the current 'I' to the length element 'dl', the vector product $dl \times r$ has magnitude $r dl \sin \theta$ and direction perpendicular to 'r' and 'dl'.

Hence, Biot–Savart law can be stated in vector form to give both the magnitude as well as direction of magnetic field due to a current element as

$$\vec{dB} = \frac{\mu}{4\pi} \frac{I (d\vec{l} \times \vec{r})}{r^3}$$

Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig. 2.3.

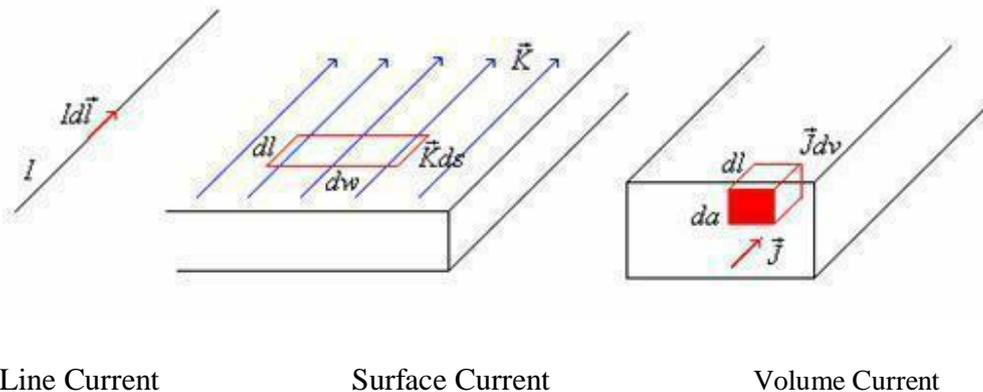


Fig. 2.3: Different types of current distributions

By denoting the surface current density as K (in amp/m) and volume current density as J (in amp/m²) we can write:

$$I d\vec{l} = \vec{K} ds = \vec{J} dv$$

(It may be noted that $I = Kdw = Jda$)

Employing Biot -Savart Law, we can now express the magnetic field intensity H. In terms of these current distributions as

$$\vec{H} = \int \frac{I d\vec{l} \times \vec{R}}{4\pi R^3} \dots\dots\dots \text{for line current} \dots\dots\dots$$

$$\vec{H} = \int \frac{K ds \times \vec{R}}{4\pi R^3} \dots\dots\dots \text{for surface current} \dots\dots\dots$$

$$\vec{H} = \int \frac{J dv \times \vec{R}}{4\pi R^3} \dots\dots\dots \text{for volume current} \dots\dots\dots$$

\vec{H} Due to infinitely long straight conductor:

We consider a finite length of a conductor carrying a current I placed along z-axis as shown in the Fig 2.4. We determine the magnetic field at point P due to this current carrying conductor.

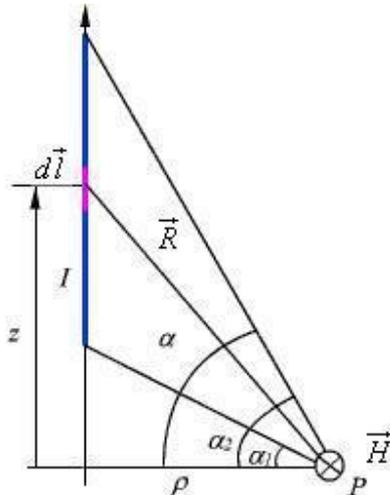


Fig. 2.4: Field at a point P due to a finite length current carrying conductor

With reference to Fig. 2.4, we find that

$$d\vec{l} = dz \hat{a}_z \text{ and } \vec{R} = \rho \hat{a}_\rho - z \hat{a}_z$$

Applying Biot - Savart's law for the current element $I d\vec{l}$ We can write,

$$d\vec{H} = \frac{I d\vec{l} \times \vec{R}}{4\pi r^3} = \frac{\rho dz \hat{a}_\phi}{4\pi [\rho^2 + z^2]^{3/2}}$$

$$\frac{z}{\rho} = \tan \alpha$$

Substituting ρ we can write,

$$\vec{H} = \int_{\alpha_1}^{\alpha_2} \frac{I}{4\pi} \frac{\rho^2 \sec^2 \alpha d\alpha}{\rho^3 \sec^3 \alpha} \hat{a}_\phi = \frac{I}{4\pi \rho} (\sin \alpha_2 - \sin \alpha_1) \hat{a}_\phi$$

We find that, for an infinitely long conductor carrying a current I , $\alpha_2 = 90^\circ$ and $\alpha_1 = -90^\circ$
Therefore

$$\vec{H} = \frac{I}{2\pi \rho} \hat{a}_\phi$$

Ampere's Circuital Law:

Ampere's circuital law states that the line integral of the magnetic field \vec{H} (circulation of H) around a closed path is the net current enclosed by this path. Mathematically,

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

The total current I_{enc} can be written as,

$$I_{enc} = \int_V \vec{J} \cdot d\vec{s}$$

By applying Stoke's theorem, we can write

$$\begin{aligned} \oint \vec{H} \cdot d\vec{l} &= \int_S \nabla \times \vec{H} \cdot d\vec{s} \\ \therefore \int_S \nabla \times \vec{H} \cdot d\vec{s} &= \int_V \vec{J} \cdot d\vec{s} \\ \therefore \nabla \times \vec{H} &= \vec{J} \end{aligned}$$

Which is the Ampere's circuital law in the point form and Maxwell's equation for magneto static fields.

Applications of Ampere's circuital law:

1. It is used to find \vec{H} and \vec{B} due to any type of current distribution.
2. If \vec{H} or \vec{B} is known then it is also used to find current enclosed by any closed path.

We illustrate the application of Ampere's Law with some examples.

\vec{H} Due to infinitely long straight conductor :(using Ampere's circuital law)

We compute magnetic field due to an infinitely long thin current carrying conductor as shown in Fig. 2.5. Using Ampere's Law, we consider the close path to be a circle of radius ρ as shown in the Fig. 4.5.

If we consider a small current element $Id\vec{l}(= Idz\hat{a}_z)$, $d\vec{H}$ is perpendicular to the plane containing both $d\vec{l}$ and $\vec{R}(= \rho\hat{a}_\rho)$. Therefore only component of \vec{H} that will be present is H_ϕ , i.e., $\vec{H} = H_\phi\hat{a}_\phi$.

By applying Ampere's law we can write,

$$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi \int_0^{2\pi} H_\phi \rho d\phi = H_\phi \rho 2\pi = I$$

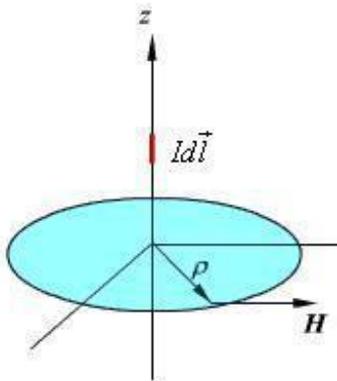


Fig. Magnetic field due to an infinite thin current carrying conductor

\vec{H} Due to infinitely long coaxial conductor :(using Ampere's circuital law)

We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current I and outer conductor carrying current $-I$ as shown in figure 2.6. We compute the magnetic field as a function of ρ as follows:

In the region $0 \leq \rho \leq R_1$

$$I_{enc} = I \frac{\rho^2}{R_1^2}$$

$$H_\phi = \frac{I_{enc}}{2\pi\rho} = \frac{I\rho}{2\pi R_1^2}$$

In the region $R_1 \leq \rho \leq R_2$

$$I_{enc} = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

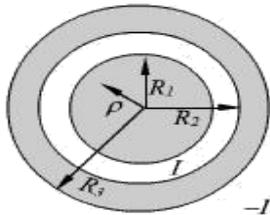


Fig. 2.6: Coaxial conductor carrying equal and opposite currents in the region

$R_2 \leq \rho \leq R_3$

$$H_\phi = \frac{I}{2\pi\rho} \frac{R_3^2 - \rho^2}{R_3^2 - R_2^2}$$

In the region $\rho > R_3$

$$I_{enc} = 0 \quad H_\phi = 0$$

Magnetic Flux Density:

In simple matter, the magnetic flux density \vec{B} related to the magnetic field intensity \vec{H} as $\vec{B} = \mu\vec{H}$ where μ called the permeability. In particular when we consider the free space $\vec{B} = \mu_0\vec{H}$ where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m².

The magnetic flux density through a surface is given by:

$$\psi = \int_S \vec{B} \cdot d\vec{s} \quad \text{Wb}$$

In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i. e. pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig. 6 (a). This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.

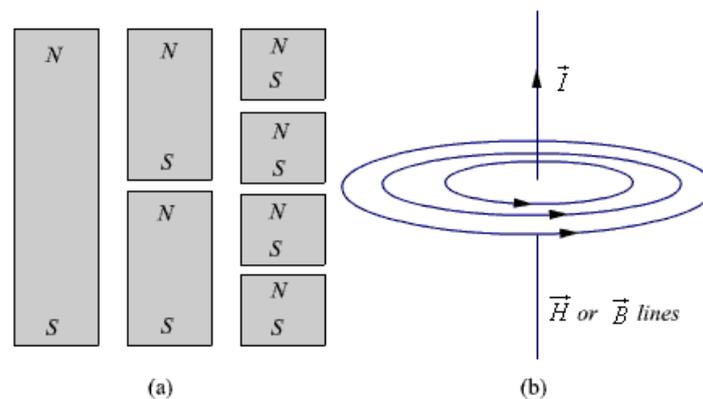


Fig. 6: (a) Subdivision of a magnet (b) Magnetic field/ flux lines of a straight current carrying conductor

Maxwell's 2nd equation for static magnetic fields:

Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig. 6 (b), we find that these lines are closed lines, that is, if we consider a closed surface, the number of flux lines that would leave the surface would be same as the number of flux lines that would enter the surface.

From our discussions above, it is evident that for magnetic field,

$$\oint_S \vec{B} \cdot d\vec{s} = 0 \dots\dots\dots \text{in integral form}$$

which is the Gauss's law for the magnetic field.

By applying divergence theorem, we can write:

$$\oint_S \vec{B} \cdot d\vec{s} = \int_V \nabla \cdot \vec{B} dv = 0$$

Hence, $\nabla \cdot \vec{B} = 0$ in point/differential form

which is the Gauss's law for the magnetic field in point form.

Magnetic Scalar and Vector Potentials:

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and write:

$$\vec{H} = -\nabla V_m$$

From Ampere's law , we know that

$$\nabla \times \vec{H} = \vec{J}$$

Therefore, $\nabla \times (-\nabla V_m) = \vec{J}$

But using vector identity, $\nabla \times (\nabla V) = 0$ we find that $\vec{H} = -\nabla V_m$ is valid only where $\vec{J} = 0$.

Thus the scalar magnetic potential is defined only in the region where $\vec{J} = 0$. Moreover, V_m in general is not a single valued function of position. This point can be illustrated as follows. Let us consider the cross section of a coaxial line as shown in fig 7.

In the region $a < \rho < b$, $\vec{J} = 0$ and $\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$

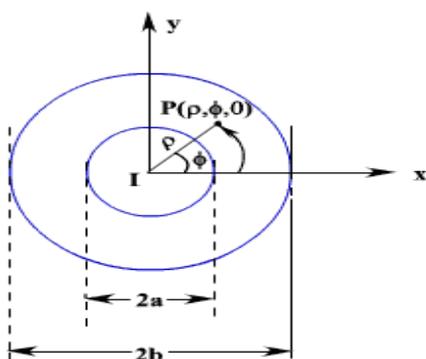


Fig. 7: Cross Section of a Coaxial Line

If V_m is the magnetic potential then,

$$\begin{aligned} -\nabla V_m &= -\frac{1}{\rho} \frac{\partial V_m}{\partial \phi} \\ &= \frac{I}{2\pi\phi} \end{aligned}$$

If we set $V_m = 0$ at $\phi = 0$ then $c = 0$ and $V_m = -\frac{I}{2\pi} \phi$

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi} \phi_0$$

We observe that as we make a complete lap around the current carrying conductor, we reach ϕ_0 again but V_m this time becomes

$$V_m = -\frac{I}{2\pi} (\phi_0 + 2\pi)$$

We observe that value of V_m keeps changing as we complete additional laps to pass through the same point. We introduced V_m analogous to electrostatic potential V .

But for static electric fields,

$$\nabla \times \vec{E} = 0 \quad \text{and} \quad \oint \vec{E} \cdot d\vec{l} = 0$$

whereas for steady magnetic field $\nabla \times \vec{H} = 0$ wherever $\vec{J} = 0$ but $\oint \vec{H} \cdot d\vec{l} = I$ even if $\vec{J} = 0$ along the path of integration.

We now introduce the vector magnetic potential which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

Since $\nabla \cdot \vec{B} = 0$ and we have the vector identity that for any vector \vec{A} , $\nabla \cdot (\nabla \times \vec{A}) = 0$, we can write $\vec{B} = \nabla \times \vec{A}$.

Here, the vector field \vec{A} is called the vector magnetic potential. Its SI unit is Wb/m. Thus if we can find \vec{A} of a given current distribution, \vec{B} can be found from \vec{A} through a curl operation. We have introduced the vector function \vec{B} and \vec{A} related its curl to \vec{B} . A vector function is defined fully in terms of its curl as well as divergence. The choice of $\nabla \cdot \vec{A}$ is made as follows.

$$\nabla \times \nabla \times \vec{A} = \mu \nabla \times \vec{H} = \mu \vec{J}$$

By using vector identity, $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

$$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}$$

Great deal of simplification can be achieved if we choose $\nabla \cdot \vec{A} = 0$.

Putting $\nabla \cdot \vec{A} = 0$, we get $\nabla^2 \vec{A} = -\mu \vec{J}$ which is vector poisson equation.

In Cartesian coordinates, the above equation can be written in terms of the components as

$$\nabla^2 A_x = -\mu J_x$$

$$\nabla^2 A_y = -\mu J_y$$

$$\nabla^2 A_z = -\mu J_z$$

The form of all the above equation is same as that of

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

for which the solution is

$$V = \frac{1}{4\pi\epsilon} \int_V \frac{\rho}{R} dv', \quad R = |\vec{r} - \vec{r}'|$$

$$\nabla \cdot \vec{A} = \mu\epsilon \frac{\partial V}{\partial t}$$

In case of time varying fields we shall see that, which is known as Lorentz condition, V being the electric potential. Here we are dealing with static magnetic field, so $\nabla \cdot \vec{A} = 0$.

By comparison, we can write the solution for A_x as

$$A_x = \frac{\mu}{4\pi} \int_V \frac{J_x}{R} dv'$$

Computing similar solutions for other two components of the vector potential, the vector potential can be written as

$$\vec{A} = \frac{\mu}{4\pi} \int_V \frac{\vec{J}}{R} dv'$$

This equation enables us to find the vector potential at a given point because of a volume current density \vec{J} .

Similarly for line or surface current density we can write

$$\vec{A} = \frac{\mu}{4\pi} \int \frac{I d\vec{l}'}{R}$$

$$\vec{A} = \frac{\mu}{4\pi} \int \frac{\vec{K} ds'}{R}$$

The magnetic flux ψ through a given area S is given by

$$\psi = \int_S \vec{B} \cdot d\vec{s} \quad \text{Substituting } \vec{B} = \nabla \times \vec{A}$$

$$\psi = \int_S \nabla \times \vec{A} \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$$

Vector potential thus have the physical significance that its integral around any closed path is equal to the magnetic flux passing through that path.

Forces due to magnetic fields

There are three ways in which the force due to magnetic fields can be experienced. The force can be

(a) Force on a charged particle:

We have $F_e = QE$

This shows that if Q is positive, F_e and E are in same direction. It is found that the magnetic force F_m experienced by a charge Q moving with a velocity u in magnetic field B is

$$F_m = Qu \times B$$

For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$$F = F_e + F_m$$

or

$$F = Q(E + u \times B)$$

This is known as Lorentz force equation.

(b) Force on a current element:

To determine the force on a current element Idl of a current carrying conductor due to the magnetic field B , we take the equation

$$J = P_e u$$

$$\text{We have } Idl = \frac{dQ}{dt} \cdot dl = dQ \cdot \frac{dl}{dt} = dQu$$

Hence

$$Idl = dQu$$

This shows that an elemental charge dQ moving with velocity u (thereby producing convection current element dQu) is equivalent to a conduction current element Idl .

Thus the force on current element is given by

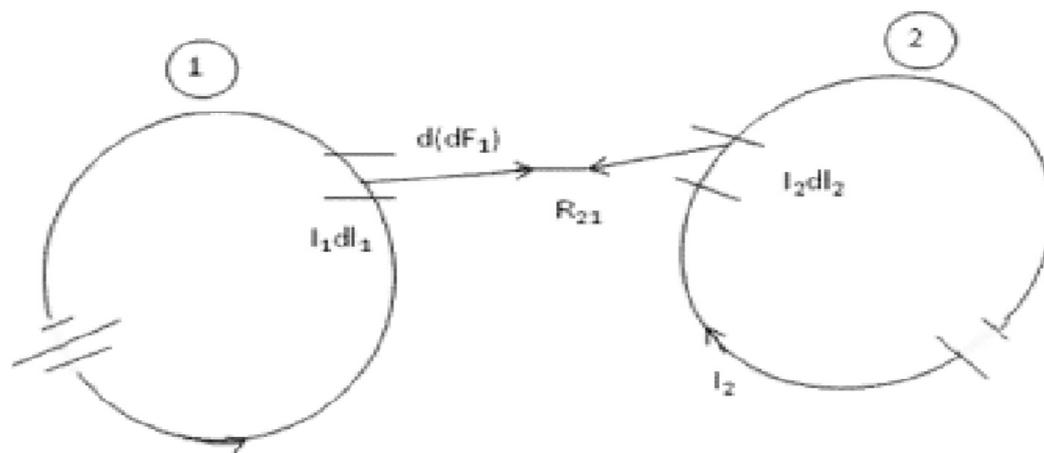
$$dF = Idl \times B$$

If the current I is through a closed path L or circuit, the force on the circuit is given by

$$F = \oint_L Idl \times B$$

(c) Force between two current elements:

Consider the force between two elements $I_1 dl_1$ and $I_2 dl_2$. According to biotsavarts law, both current elements produce magnetic fields. Force $d(F_1)$ on element $I_1 dl_1$ due to field dB_2 produced by element $I_2 dl_2$ as shown in figure below:



$$d(dF_1) = I_1 dl_1 \times dB_2$$

But from biot Savarts law

$$dB_2 = \frac{\mu_0 I_2 dl_2 \times a_{R21}}{4\pi R_{21}^2}$$

Hence

$$d(dF_1) = \frac{\mu_0 I_1 dl_1 \times (I_2 dl_2 \times a_{R21})}{4\pi R_{21}^2}$$

This equation is the law of force between two current elements.

$$\text{We have } F_{12} = \frac{\mu_0 I_1 I_2 \times a_{R21}}{4\pi} \int_{L_1} \int_{L_2} \frac{dl_1 \times (dl_2 \times a_{R21})}{R_{21}^2}$$

Inductance:

Inductance is the ability of the material to hold energy in form of magnetic field.

L , I are inductance of material and current flowing in the material.

$$E = \frac{1}{2} LI^2$$

$$\text{Inductance, } L = \frac{\text{Total flux linking current } I}{\text{current } (I)}$$

'B' is induced by I

$$\therefore \phi = \int_S \vec{B} \cdot d\vec{s}$$

Total Flux depends on no of turns

Flux linking for n turns is ' $N\phi$ '.

$$\therefore L = \frac{\lambda}{I} \quad \text{---} \quad \lambda = N\phi \text{ (depending on condition i.e total Flux linking the current)}$$

Inductance of a solenoid:

In the application of ampere's law to solenoid we found that

$$B = \frac{\mu NI}{l} \text{ Tesla}$$

$$\therefore \phi = B.A = \frac{\mu NIA}{l}$$

With in a loop of N turns, the flux is linking the current N times.

$$\begin{aligned} \therefore \text{Total flux linking } I &= N\phi \\ &= \frac{\mu N^2 IA}{l} \end{aligned}$$

$$L = \frac{\lambda}{I} = \frac{\mu N^2 A}{l}$$

Some times inductors are given for unit length as well

$$\therefore \frac{l}{l} = \mu \left(\frac{N}{l} \right)^2 . A$$

Inductance of coaxial cable:

- The total flux linking the inner and outer conductors is same as the flux in the conductor.

$$H = \frac{I}{2\pi r} \text{ (A/m)}$$

$$B = \frac{\mu I}{2\pi r} \text{ (Wb/m}^2\text{)}$$

Here flux density is differing with radius

$$\therefore \phi = \int B . d\bar{s}$$

$$\therefore \phi = \int \frac{\mu I}{2\pi r} . ds$$

$$d\bar{s} = dr dz$$

$$\phi = \int_{z=0}^L \int_{r=a}^b \frac{\mu I}{2\pi r} . dr dz$$

$$\phi = \frac{\mu I l}{2\pi} \int_a^b \frac{dr}{r}$$

$$\Rightarrow \lambda = \frac{\mu I l}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\therefore L = \frac{\lambda}{I} = \frac{\mu l}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\frac{L}{l} = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$$

Where μ is the permeability of medium used b/w inner and outer cores.

Also there is current flowing even inside the inner core.

$$= \frac{\mu l \pi a^2}{8\pi} = \frac{\mu l}{8\pi}$$

$$\therefore \frac{L_{int}}{l} = \frac{\mu}{8\pi} (H)$$

$$\frac{L_{ext}}{l} = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) (H/m)$$

Here μ is permeability of conductor

$$\frac{\text{Total inductance}}{\text{Length}} = \frac{L_{ext}}{l} + \frac{L_{int}}{l}$$

$$= \frac{\mu_1}{2\pi} \ln\left(\frac{b}{a}\right) + \frac{\mu_2}{2\pi}$$

$$L = \frac{\Lambda}{I} = \frac{N\phi}{I}$$

To further illustrate the concept of inductance, let us consider two closed loops C_1 and C_2 as shown in the figure 8, S_1 and S_2 are respectively the areas of C_1 and C_2 .

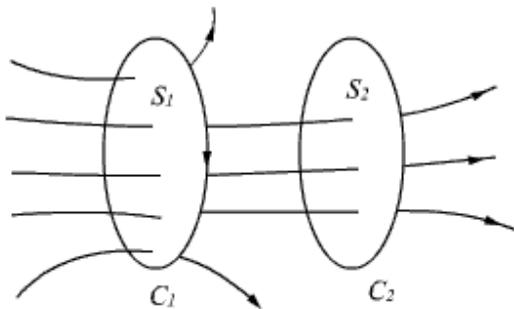


Fig:8

If a current I_1 flows in C_1 , the magnetic flux B_1 will be created part of which will be linked to C_2 as shown in Figure 8:

$$\phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2$$

In a linear medium, ϕ_{12} is proportional to I_1 . Therefore, we can write

$$\phi_{12} = L_{12} I_1$$

where L_{12} is the mutual inductance. For a more general case, if C2 has N_2 turns then

$$\Lambda_{12} = N_2 \phi_{12}$$

and $\Lambda_{12} = L_{12} I_1$

or $L_{12} = \frac{\Lambda_{12}}{I_1}$

i.e., the mutual inductance can be defined as the ratio of the total flux linkage of the second circuit to the current flowing in the first circuit.

As we have already stated, the magnetic flux produced in C1 gets linked to itself and if C1 has N_1 turns then $\Lambda_{11} = N_1 \phi_{11}$, where ϕ_{11} is the flux linkage per turn.

Therefore, self inductance

$$L_{11} (\text{or } L \text{ as defined earlier}) = \frac{\Lambda_{11}}{I_1}$$

As some of the flux produced by I_1 links only to C1 & not C2.

$$\Lambda_{11} = N_1 \phi_{11} > N_2 \phi_{12} = \Lambda_{12}$$

Further in general, in a linear medium, $L_{12} = \frac{d\Lambda_{12}}{dI_1}$ and $L_{11} = \frac{d\Lambda_{11}}{dI_1}$

Magnetic energy or Energy stored in Magnetic Field:

So far we have discussed the inductance in static forms. In earlier chapter we discussed the fact that work is required to be expended to assemble a group of charges and this work is stated as electric energy. In the same manner energy needs to be expended in sending currents through coils and it is stored as magnetic energy. Let us consider a scenario where we consider a coil in which the current is increased from 0 to a value I . As mentioned earlier, the self inductance of a coil in general can be written as

$$L = \frac{d\Lambda}{di} = N \frac{d\phi}{di}$$

or $L di = N d\phi$

If we consider a time varying scenario,

$$L \frac{di}{dt} = N \frac{d\phi}{dt}$$

We will later see that $N \frac{d\phi}{dt}$ is an induced voltage.

$\therefore v = L \frac{di}{dt}$ is the voltage drop that appears across the coil and thus voltage opposes the change of current.

Therefore in order to maintain the increase of current, the electric source must do an work against this induced voltage.

$$\begin{aligned} dW &= vi dt \\ &= Li di \end{aligned}$$

$$W = \int_0^I Li di = \frac{1}{2} LI^2 \quad (\text{Joule})$$

which is the energy stored in the magnetic circuit.

We can also express the energy stored in the coil in term of field quantities.

For linear magnetic circuit

$$W = \frac{1}{2} \frac{N\phi}{l} I^2 = \frac{1}{2} N\phi I$$

Now,

$$\phi = \int_s \vec{B} \cdot d\vec{S} = BA$$

where A is the area of cross section of the coil. If l is the length of the coil

$$NI = Hl$$

$$\therefore W = \frac{1}{2} HBAI$$

Al is the volume of the coil. Therefore the magnetic energy density i.e., magnetic energy/unit volume is given by

$$W_m = \frac{W}{Al} = \frac{1}{2} BH$$

In vector form

$$W_m = \frac{1}{2} \vec{B} \cdot \vec{H} \quad \text{J/m}^3$$

is the energy density in the magnetic field.