# UNIT- III

# Noise

Types of Noise

Time domain representation of Narrow band Noise

filtered White Noise

Quadrature Representation of Narrowband noise plus sine wave

Signal to Noise ratio

probability of error

Noise Equivalent bandwidth

Effective Noise temperature

Noise figure

Baseband systems with channel Noise

Performance analysis of AM, DSB-SC, SSB-SC, FM, PM in the presence of Noise

Illustrative problems.

## Definition of Noise:-

In Electrical terms, noise may be defined as an unwanted form of energy which tend to interfere with the proper reception & reproduction of transmitted signals. There are numerous ways of classifying noise. It may be sub divided according to type, source, effect (or) relation to receiver. The most convenient way is to divide noise in two broad groups.
(i) External Noise
(ii) Internal Noise

Noise

- External Noise
  - Extraterrestrial Noise
    - Solar Noise
    - Cosmic Noise
  - Atmospheric Noise
  - Industrial Noise
- Internal Noise
  - Shot Noise
  - Partition Noise
  - Low frequency Noise (or) flicker Noise
  - High frequency Noise (or) Transit-time Noise
  - Thermal Noise

## External Noise:-

External Noise may be defined as that type of noise which is generated external to a communication system i.e whose sources are external to the communication system.

## Atmospheric Noise:-

Atmospheric Noise which is also called static, is produced by lightning discharges in thunderstorms and other natural electrical disturbances which occur in atmosphere.

Atmospheric Noise contains spurious radio signals which are distributed over a wide frequency range. Due to this reason, at any given receiving point, the receiving antenna picks up not only the required signal but also the static from all the thunder storms. Thus atmospheric noise becomes less severe at frequencies above about 30MHz

Solar Noise :-

Solar Noise is the electrical noise eminating from the sun. under steady conditions, there is a regular radiation of noise from the sun. This radiation of noise from the sun is due to the fact that sun is a big body at an extremely high temperature and it radiates electrical energy in the form of noise over a very wide frequency spectrum including also the frequency spectrum which is occupied by radio communication.

Cosmic Noise:-

Distant stars can also be considered suns. These distant stars have high temperatures and therefore radiate noise in the same manner as the sun. The noise received from these distant stars is thermal noise and is distributed almost uniformly over the entire sky

Industrial Noise:-

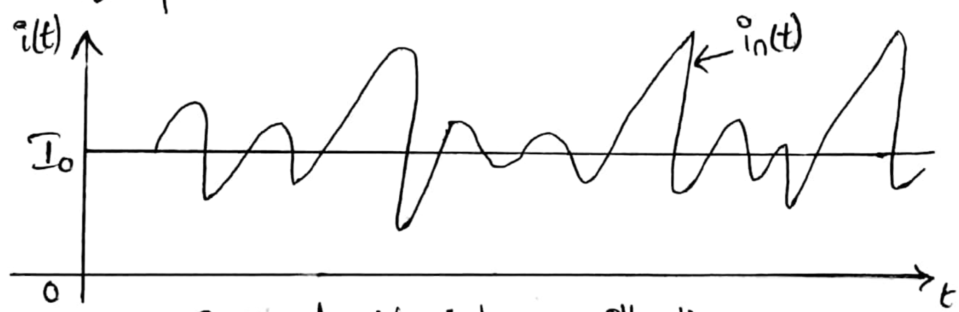
The industrial (or) man-made noise is that type of noise which is produced by such sources as automobiles and aircraft ignition, electrical motors, switch gears and leakage from high voltage transmission lines and several other heavy electrical equipments. Industrial noise in industrial areas, dense populated areas is much stronger than all other sources of noise in the frequency range extending from about 1MHz to 600MHz.

## Internal Noise :-

Internal Noise is that type of noise which is generated internally & within the communication system & receiver.

## Shot Noise :-

It arises in active devices due to random behaviour of charge carriers. In electron tubes, shot noise is generated due to random emission of electrons from cathodes, whereas in semiconductor devices shot noise is generated due to random diffusion of minority carriers & simply random generation and recombination of electron-hole pairs.




Current variation with time

Total current $i(t)$ can be expressed as

$$\boxed{i(t) = I_0 + i_n(t)}$$

$I_0$ is the mean

$i_n(t)$ is the (fluctuating) shot-noise current

Partition Noise :-

Partition noise is generated in a circuit when a current has to divide between two & more paths. This means that the partition noise results from the random fluctuations in the division. For partition Noise, the spectrum is a flat spectrum.

Low frequency Noise (or) flicker Noise :-

At low frequencies, a particular type of noise appears. The power spectral density of this noise increases as the frequency decreases. This noise is called as flicker noise (or) (1/f) noise. The power density spectrum of the flicker noise is inversely proportional to frequency, Mathematically,

$$S(\omega) \propto \frac{1}{f}$$

Therefore, the flicker noise becomes significant at very low frequencies, generally below a few KHz

Transit-time Noise (or) High frequency Noise :-

In semiconductor devices, when the transit-time of charge carriers crossing a junction is comparable with the time period of the signal, some charge-carriers diffuse back to the source & emitters. This process gives rise to an input admittance in which the conductance component increases with frequency. Because of this conductance increases with frequency, the power spectral density will also increase.

## Noise:-

The term noise is used customarily to designate unwanted waves that tend to disturb the transmission & Processing of signals in communication systems, and over which we have incomplete control. The sources of noise may be external to the system & internal to the system. The second category includes an important type of noise that arises due to spontaneous fluctuations of current & voltage in electrical circuits.

## Thermal Noise

It is the name given to the electrical noise arising from the random motion of electrons in a conductor. The mean-square law of the thermal noise voltage appearing across the terminals of a resistor, measured in a bandwidth of $\Delta f$ hertz is, for all practical purposes given by

$$E[V_{TN}^2] = 4KTR\,\Delta f \text{ volts}^2$$

Where K is Boltzmann's constant $= 1.38 \times 10^{-23} J/^{o}K$

T is absolute temperature in $^{o}K$

R is resistance in ohms.

The analysis of thermal noise is based on Kinetic theory which shows that the temperature of a particle is a way of expressing its KE. Thus noise power generated in a resistor is directly proportional to its absolute temperature. Thermal noise is also known as a resistor noise (or) Johnson noise.

Noise calculations involve the transfer of power, and so we find that the use of the maximum-power transfer theorem is applicable.

## Available Noise Power:-

The maximum amount of power received from the source is termed as the available power.

According to maximum power transfer theorem, maximum power is transferred from source to load if source resistance equals the load resistance & maximum power is transferred from source to load if source impedance is complex conjugate of load impedance.

Let $Z_s$ be the source impedance and $Z_L$ be the load impedance, Now if

$$Z_s = R_s + jX_s \longrightarrow (1)$$

Then

$$Z_L = Z_s^* = R_s - jX_s \longrightarrow (2)$$

The root mean square current (r.m.s) passing through the load is given in terms of noise voltage $V_n$ as

$$I_{r.m.s} = \frac{\sqrt{\overline{V_n^2}}}{Z} \quad \text{But, } Z = 2R_s$$

$$\Rightarrow I_{r.m.s} = \sqrt{\overline{V_n^2}} / 2R_s$$

The available power, $P = I^2R$

$$= I_{r.m.s}^2 \times R_s$$

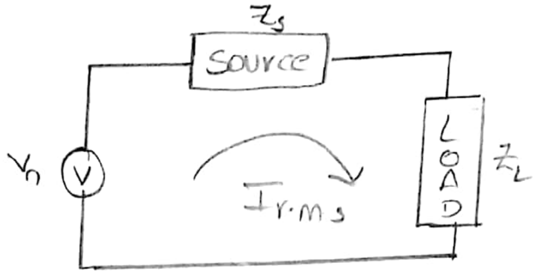
$$= \frac{\overline{V_n^2}}{4R_s^2} \times R_s$$

$$= \overline{V_n^2} / 4R_s$$

$$= \frac{P_{nr}}{4R_s}$$

Here $P_{nr}$ is normalized power

The complete scenario is shown in the following fig




figure

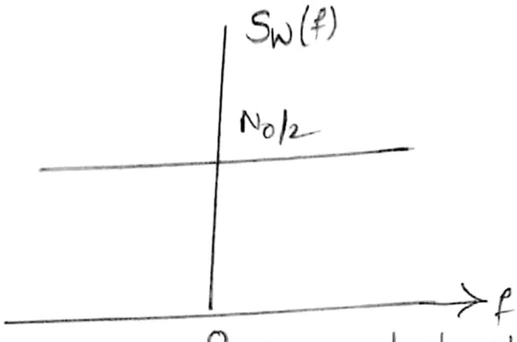
The total noise power supplied by noise voltage $V_n$ is $\frac{P_n}{2R_s}$. From this power $\frac{P_n}{4R_s}$ which is exactly half of noise power is available across the load, the left out power is dissipated across the resistance R.

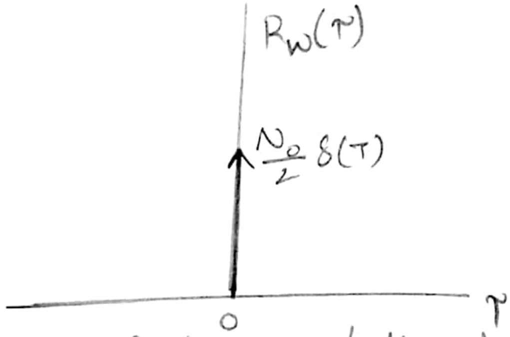
White Noise:-

The noise analysis of communication systems is customarily based on an idealized form of noise called white noise, the power spectral density of which is independent of the operating frequency. The adjective white is used in the sense that white light contains equal amounts of all frequencies within the visible band of electromagnetic radiation. The power spectral density of white noise $w(t)$ as

$$S_w(f) = \frac{N_0}{2} \longrightarrow (1)$$

Where the factor ½ has been included to indicate that half the power is associated with +ve frequency and half with -ve frequency as illustrated in fig (a). The dimensions of $N_0$ are in watts per hertz. The parameter $N_0$ is usually referenced to the i/p stage of the receiver of a communication system. It may expressed as $N_0 = KT_e \longrightarrow (2)$




(a) Power Spectral density (b) Autocorrelation function

Characteristics of White Noise

Where K is Boltzmann's constant & Te is the equivalent noise temperature of the receiver. The equivalent noise temperature of a system is defined as the temp at which a noisy resistor has to be maintained such that, by connecting the resistor to the i/p of a noiseless version of the system, it produces the same available noise power at the o/p of the system as that produced by all the sources of noise in the actual system. The important feature of the equivalent noise temperature is that it depends only on the parameters of the system.

Since the autocorrelation function is the inverse Fourier transform of the power spectral density, it follows that for white noise

$$R_W(\tau) = \frac{N_o}{2}\delta(\tau) \longrightarrow (3)$$

The utility of a white noise process is ||el to that of an impulse function & delta function in the analysis of linear systems.

White noise is a wide sense stationary noise process. White noise has power density spectrum that has constant value at all frequencies i.e

Power density, $S_{NN}(w) = \frac{N_o}{2}$

Where $N_o$ is a real +ve constant.

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## Noise Equivalent Bandwidth:-

Noise equivalent bandwidth is calculated by replacing an arbitrary LPF having a transfer function H(f) by an identical LPF having zero-frequency response and bandwidth B Hz.

Therefore, average noise power (N) of a system having a zero mean white noise and power spectral density $\left(\frac{N_0}{2}\right)$ connected to i/p of an arbitrary LPF having transfer function H(f) is given as

$$N = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

$$= N_0 \int_{0}^{\infty} |H(f)|^2 \, df \longrightarrow (1)$$

Now, if the arbitrary LPF is replaced by an ideal low pass filter having bandwidth B Hz and zero frequency response, then

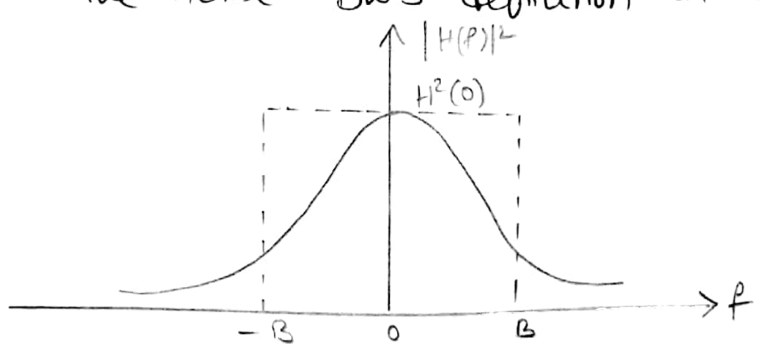
$$N = N_0 B H^2(0) \longrightarrow (2)$$

from equations (1) & (2), we can write

$$N_0 \int_0^{\infty} |H(f)|^2 df = N_0 B H^2(0)$$

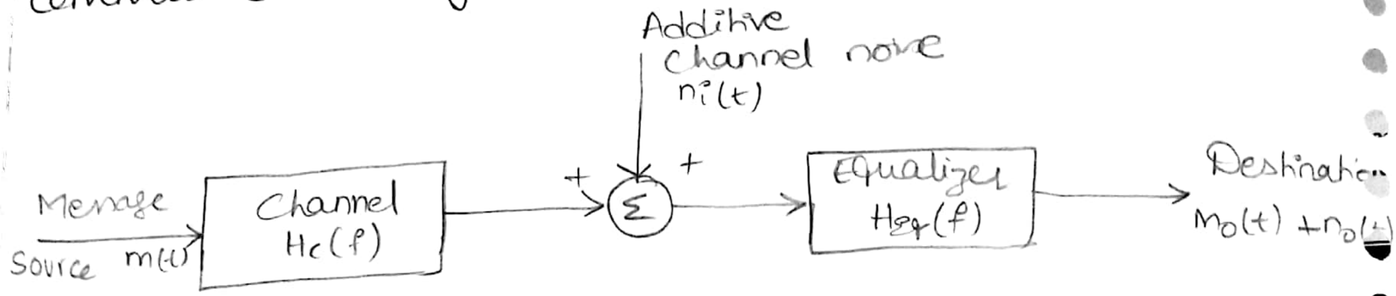
$$B = \frac{\int_0^{\infty} |H(f)|^2 \, df}{H^2(0)}$$

figure shows the noise BW's definition in a graphical representation.



## Base band signal Transmission with Noise:

The basic block diagram of a baseband communication system is shown in figure




The baseband signal m(t) is to be transmitted over a baseband channel. Consider, the transfer function of the channel is $H_c(f)$ and the linear distortions due to the channel are removed by an equalizer with a transfer function

$$H_{eq}(f) = K[H_c(f)]^{-1} \exp(-j2\pi f t_d) \rightarrow (1)$$

So, the signal obtained at the destination point is distortionless i.e $m_o(t) = Km(t - t_d) \rightarrow (2)$

The channel also corrupts, the signal with additive noise $n_i(t)$ and which produces an additive noise component $n_o(t)$ at the destination.

The signal m(t) and front-end noise $n_i(t)$ are random process with the following properties

1. m(t) is a stationary, zero means low-pass random process band limited to '$f_m$' with a PSD function $G_m(f)$
2. $n_i(t)$ is a stationary, zero means gaussian random process with a PSD function $G_{ni}(f)$
3. m(t) & $n_i(t)$ are independent.

SNR at the o/p of a baseband system:

The signal quality at the o/p of analog communi-cation systems is usually measured by the average signal power to noise power ratio as

$$\left(\frac{S}{N}\right)_d = \frac{E[m_o^2(t)]}{E[n_o^2(t)]} \rightarrow (1)$$

This ratio ranges from

10dB for barely intelligence voice signals

30dB " telephone quality voice signals

60dB " high-fidelity audio "

For a baseband communication system, we can assume that,

$$H_{eq}(f)H_c(f) = \begin{cases} K\exp(-j2\pi f t_d) & \text{for } |f|<f_m \\ 0 & \text{elsewhere} \end{cases}$$

Then we have,

$$x_o(t) = Kx(t-t_d) \text{ and}$$

$$E\{m_o^2(t)\} = K^2 E\{m^2(t-t_d)\} = K^2 \int_{-f_m}^{f_m} G_m(f)df \rightarrow (2)$$

The average noise power at the o/p is calculated as

$$E\{n_o^2(t)\} = \int_{-\infty}^{\infty} G_{n_o}(f)\,df = \int_{-f_m}^{f_m} G_{n_i}(f)|H_{eq}(f)|^2 df$$

$$= \int_{-f_m}^{f_m} K^2 \frac{G_{n_i}(f)\,df}{|H_c(f)|^2} \rightarrow (3)$$

The o/p signal to noise ratio is given by,

$$\left(\frac{S}{N}\right)_d = K^2 \int_{-f_m}^{f_m} G_m(f)df \Bigg/ \int_{-f_m}^{f_m} K^2 \frac{G_{n_i}(f)\,df}{|H_c(f)|^2}$$

$$\left(\frac{S}{N}\right)_d = \frac{\int_{-f_M}^{f_M} G_m(f)\, df}{\int_{-f_M}^{f_M} G_{n_i}(f)\left[|H_c(f)|^2\right]^{-1} df} \longrightarrow (4)$$

## Noise in a Continuous Wave Communication Systems:-

The block diagram of an ideal continuous wave communication system as shown in figure. The signal $m(t)$ is applied at the transmitting end with a PSD $G_m(f)$ band limited to $f_m$ Hz.

Then the transmitter o/p is assumed as,

$$m_c(t) = A(t)\cos[w_c t + \phi(t)] \longrightarrow (1)$$

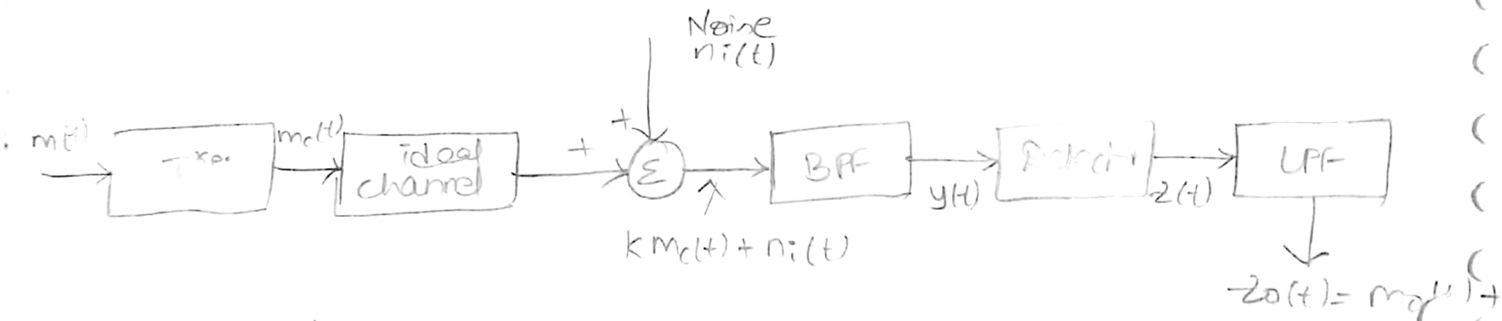



figure: Continuous wave communication system

Where $x_c(t)$ is modulated signal with a BW '$B_T$'.

At the o/p of the channel, the s/n is accompained by additive noise with a PSD of $G_{n_i}(f)$

At the receiver end, an ideal BPF with a BW $B_T$ which is same as the BW of $m_c(t)$. The BPF passes $m(t)$ without any distortion, but limits the amount of out of band noise that reaches the detector.

The i/p to the detector consists of a signal components $km_c(t)$ and noise component $n(t)$ which is filtered version of $n_i(t)$. The bandpass noise $n(t)$ can be represented as

$$n(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t \longrightarrow (2)$$

i/p to detector can be expressed as

$$y(t) = Kx_c(t) + n(t)$$

$$= R_y(t)\cos[\omega_c t + \theta_y(t)] \longrightarrow (3)$$

$$= y_c(t)\cos\omega_c t - y_s(t)\sin\omega_c t \longrightarrow (4)$$

The o/p of the detector depends on type of detector used in the process i.e $z(t)$

$$z(t) = \begin{cases} K_1 y_c(t) & \text{— synchronous detector} \\ K_2 R_y(t) & \text{— Envelope detector} \\ K_3 \theta_y(t) & \text{— Phase detector} \\ K_4 (d(\theta_y(t))/dt) & \text{— Frequency detector} \end{cases}$$

Where $K_1, K_2, K_3$ and $K_4$ are gain constants which are assumed as unity.

The detected o/p is passed through an ideal LPF, band limited to $f_m$ the o/p can be represented as

$$z_o(t) = m_o(t) + n_o(t) \longrightarrow (5)$$

In most of the practical systems the front-end noise can be assumed as white gaussian noise with PSD of

$$G_{nT}(f) = \eta/2 \text{ watt/Hz} \longrightarrow (6)$$

The i/p signal quality is measured by the S/N -ratio at the detector as

$$\left(\frac{S}{N}\right)_i = \frac{K^2 E\{m_c^2(t)\}}{B_T \eta} \longrightarrow (7)$$

Every communication system experiences noise, we can say that the SNR is always finite. The

Comparison of S/N ratio at the i/p & the o/p of any communication system gives the noise indication of that n/w

## Noise calculation in Amplitude Modulation:-

The performance of the AM system is analyzed using Envelope detector. This type of demodulation yields satisfactory performance as long as the signal power at the i/p of the receiver is considerably higher than that in band noise power.

① Input signal Power ($S_i$)

In amplitude modulation system, a carrier is accompanied with the two upper & lower sidebands. Let $n_i(t)$ be the additive noise signal, then the i/p to the detector is expressed as,

$$s(t)_{AM} = A[1 + k_a m(t)] \cos\omega_c t + n_i \longrightarrow ①$$

The i/p signal power $S_i$ is given by

$S_i$ = Mean square value of carrier + Mean square value of sidebands

$$= \frac{A^2}{2}\left[1 + k_a^2 \overline{m^2(t)}\right] \longrightarrow ②$$

② Input Noise power ($N_i$)

The noise signal $n_i(t)$ at the i/p of the detector is $n_c(t)$ with the power spectrum density is given as,

$$S_{n_i}(\omega) = S_{n_c}(\omega) \longrightarrow ③$$

As the AM system, contains two sidebands and $S_{n_c}(\omega)$ is equal to the PSD of the channel. i.e

$$S_{n_c}(\omega) = \cancel{\eta}\,\eta/2 \longrightarrow ④$$

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The i/p noise power is given by

$$N_i = S_{ni}(\omega)(B.W) = \frac{\eta}{2}(2f_m) \quad (\because S_{ni}(\omega) = S_{nc}(\omega))$$

$$\therefore N_i = 2\eta f_m \longrightarrow (5)$$

Output power:-

The o/p of the envelope detector will be the envelope of the AM signal $s(t)_{AM}$. Therefore, for calculating the detected o/p, we are required to find the envelope of $s(t)_{AM}$.

Substituting the bandpass noise in the eq (1), we get

$$s(t)_{AM} = A[1+k_a m(t)]\cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t \longrightarrow (6)$$

Simplifying the above equation using trignometric propert we get,

$$s(t)_{AM} = A(t)\cos(\omega_c t + \phi(t)) \longrightarrow (7)$$

where $A(t)$ = Randomly time varying amplitude

$\phi(t)$ = Phase angle of $s(t)_{AM}$

i.e $$A(t) = \sqrt{\{A[1+k_a m(t)] + n_c(t)\}^2 + n_s^2(t)} \longrightarrow (8)$$

$$\phi(t) = \tan^{-1}\left[\frac{n_s(t)}{A[1+k_a m(t)+n_c(t)}\right] \longrightarrow (9)$$

Large Noise:

In this case, the parameter $n_i(t)$ is much greater than $A[1+k_a m(t)]$ i.e $n_i(t) >> A + k_a m(t)$

Due to this condition the noise term dominates the signal terms, hence the performance of system with larger noise is different as compared with the case of low noise. Therefore the equation (8) for A(t) can now be expressed as

$$A(t) = \sqrt{\{A + k_a m(t) + n_c(t)\}^2 + n_s^2(t)}$$

$$= \sqrt{(A + k_a m(t))^2 + n_c^2(t) + 2n_c(t)[A + k_a m(t)] + n_s^2(t)} \rightarrow (10)$$

In the above equation, the term $(A + k_a m(t))^2$ is much small compared to other terms in the equation, hence it can be neglected

$$A(t) = \sqrt{n_c^2(t) + n_s^2(t) + 2n_c(t)[A + k_a m(t)]}$$

$$= \sqrt{n_c^2(t) + n_s^2(t)\left[1 + \frac{2n_c(t)(A + k_a m(t))}{n_c^2(t) + n_s^2(t)}\right]}$$

Putting the noise component as

$$R(t) = \sqrt{n_c^2(t) + n_s^2(t)} \quad \& \quad \theta(t) = \tan^{-1}\left[\frac{n_c(t)}{n_s(t)}\right]$$

$$A(t) = \sqrt{R^2(t)\left[1 + 2(A + k_a m(t))\frac{n_c(t)}{R^2(t)}\right]}$$

$$= R(t)\left[1 + \frac{2[A + k_a m(t)]}{R(t)}\cos\theta(t)\right]^{1/2}$$

The term R(t) ie noise component is much greater than signal component $2[A + k_a m(t)]$ hence equation (11) can be written as

$$A(t) = R(t)\left[1 + \frac{1}{2} \times \frac{2[1 + k_a m(t)]A}{R(t)}\cos\theta(t)\right]$$

$$= R(t) + A[1 + k_a m(t)]\cos\theta(t) \rightarrow (12)$$

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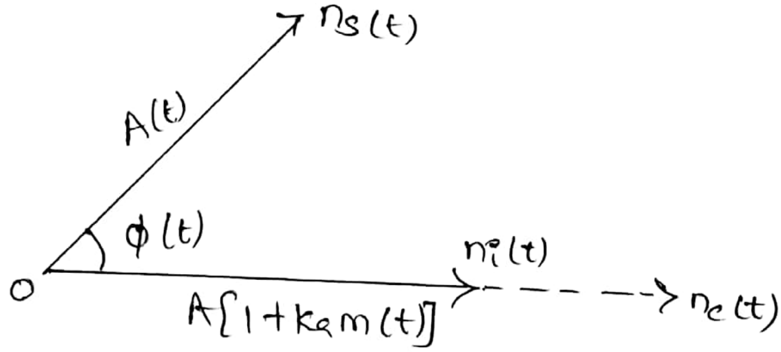
From equation (12) we can conclude the envelope $A(t)$ appearing at o/p of the detector contains no separate term related to modulating signal $\{k_a m(t)\}$. The envelope $A(t)$ has both signal & noise components. The noise performance depends on the relative magnitudes of signal & noise, if we are considering that noise is much smaller than s/n i.e

$$n_i(t) << [A + k_a m(t)] \longrightarrow (1)$$

A phasor representation of envelope $A(t)$ is shown in figure below. The noise component $n_c(t)$ is shown to be in phase with signal $A[1+k_a m(t)]$ whereas $n_s(t)$ is in phase quadrature

But
$$n_i(t) << A[1+k_a m(t)]$$
$$n_s(t) << A[1+k_a m(t)]$$




From the phasor diagram, both $n_s(t)$ & $\phi(t)$ are much smaller and assumed to be zero. When $\phi(t)=0$, then the envelope becomes,

$$A(t) = A[1+k_a m(t)] + n_c(t) \longrightarrow (2)$$

∴, the o/p of the envelope detector contains a useful signal $m(t)$ & a noise component $n_c(t)$. The signal power $S_o$ & noise power $N_o$ at the o/p of the detector may be calculated as

(i) output signal power ($S_o$)

The mean square value of the useful s/n $m(t)$ mathematically is $S_o = \overline{m^2(t)}$ $\longrightarrow$ (13)

(ii) output Noise power ($N_o$)

The noise signal $n_o(t)$ at the o/p of the detector is $n_c(t)$ with the power spectrum density given

$$S_{no}(\omega) = S_{nc}(\omega) \longrightarrow (14)$$

As the AM system contains both sidebands

$$S_{no}(\omega) = \eta \longrightarrow (15)$$

The output noise power is given by

$$N_o = S_{no}(\omega)(B.W) = \eta(2f_m) = 2\eta f_m$$

Then the figure of merit ($\alpha$) for the AM s/m is given by

$$\alpha = \frac{S_o/N_o}{S_i/N_i} = \frac{\dfrac{K_a^2 A^2 \overline{m^2(t)}}{2\eta f_m}}{\dfrac{\frac{A^2}{2}\left[1+K_a^2 \overline{m^2(t)}\right]}{2\eta f_m}}$$

$$\alpha = \frac{2K_a^2 \overline{m^2(t)}}{\left[1+K_a^2 \overline{m^2(t)}\right]} \longrightarrow (16)$$

From equation (16), it is clear that the performance of the noise improves with reduction of carrier amplitude A and reaches maximum when A = 0.

Thus, the best noise performance is achieved

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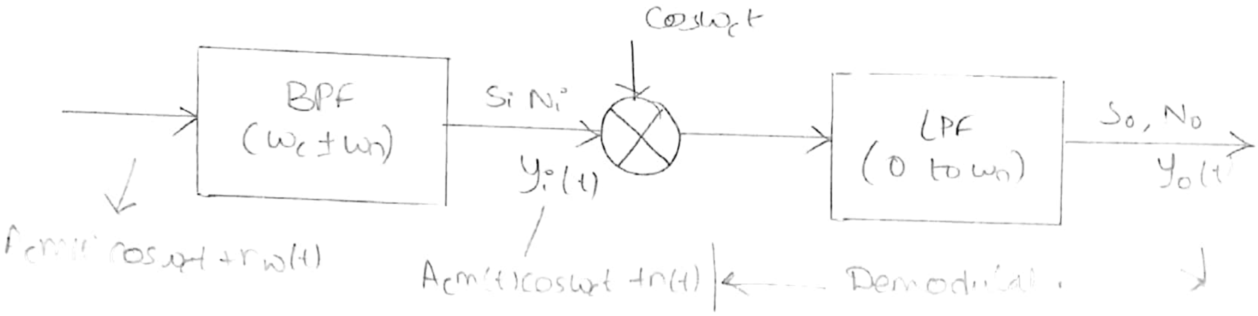
when the carrier amplitude A is equal to the maximum value of m(t) ie for 100% modulation

## Noise in DSBSC:—

A DSB-SC receiver can be modeled as a BPF followed by a coherent (synchronous detector) as shown in figure. Assume that the BPF is ideal with unity gain and that its passband corresponds to the modulated signal frequency range




Coherent (synchronous) Detector

The (modulated) signal plus white noise enters the receiver i/p. The signal is unchanged upon filtering whereas the white noise $n_w(t)$, gets converted to band pass noise $n(t)$. The o/p of the BPF is the i/p to the demodulator $y_i(t)$, and is given by

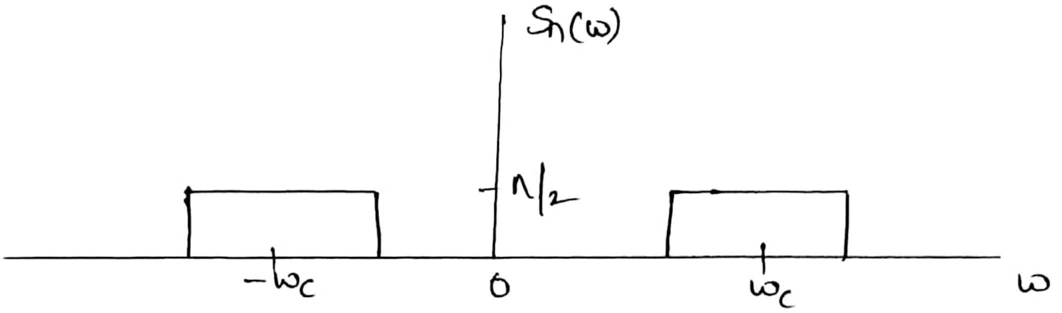
$$y_i(t) = A_c m(t) \cos\omega_c t + n(t) \rightarrow (1)$$

Clearly from equation (1), the i/p signal component is $S_i(t) = A_c m(t) \cos\omega_c t$. Hence, the signal power at the demodulator i/p is given by the mean square value of $S_i(t)$;

$$S_i = \overline{\{S_i(t)\}^2} = \overline{[A_c(m(t)\cos\omega_c t]^2} = \frac{1}{2}A_c^2\overline{m^2(t)} \rightarrow (2)$$

the factor is due to the mean square value of the cosine term

From equation ① , the demodulator i/p noise power $N_i$ is the mean square value of the bandpass noise $n(t)$ since the band of the bandpass filter is from $\omega_c - \omega_m$ to $\omega_c + \omega_m$, the PSD of $n(t)$ is same as in below figure




PSD of bandpass white noise

Hence

$$N_i = \overline{n^2(t)} = 2\int_{f_c-f_m}^{f_c+f_m} S_n(f)\,df = 2\int_{f_c-f_m}^{f_c+f_m} \frac{\eta}{2}\,df = 2\eta f_m \quad \rightarrow ③$$

To find the o/p signal power $S_o$, and o/p noise power $N_o$, we have to obtain the expression for the o/p of the low pass filter, $y_o(t)$ clearly,

$$y_o(t) = LPF\{y_i(t) \times \cos\omega_c t\} \quad \rightarrow ④$$

Where LPF has been used to denote the LPF operation for $y_i(t)$, we use equation ① & write the bandpass noise $n(t)$ in term of its quadrature representation. Thus equation ④ becomes

$$y_o(t) = LPF\{[A_c m(t)\cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t] \times [\cos\omega_c t]\}$$

$$= LPF\{[A_c m(t)\cos^2\omega_c t + n_c(t)\cos\omega_c^2 t - n_s(t)\sin\omega_c t\cos\omega_c t]\}$$

$$= LPF\left[\frac{A_c}{2}m(t) + \frac{A_c}{2}m(t)\cos 2\omega_c t + \frac{n_c(t)}{2} + \frac{n_c(t)}{2}\cos 2\omega_c t - \frac{n_c}{2}\sin 2\omega_c t\right]$$

The o/p of the LPF is

$$y_0(t) = \frac{A_c}{2} m(t) + \frac{n_c(t)}{2} \rightarrow (5)$$

From equation (5), the useful o/p s/n component is $S_0(t) = \frac{A_c}{2} m(t)$ & the noise component is $n_0(t) = \frac{n_c(t)}{2}$

The noise at the detector o/p involves the in-phase component only

Hence, the o/p signal power is

$$S_i = \overline{S_0^2(t)} = \overline{\left[\frac{A_c}{2} m(t)\right]^2} = \frac{A_c^2}{4} \overline{m^2(t)} \rightarrow (6)$$

$$N_0 = \overline{n_0^2(t)} = \overline{\left[\frac{n_c}{2}\right]^2} = \frac{1}{4} \overline{n_c^2(t)} = \frac{1}{4} \overline{n_c^2(t)} \rightarrow (7)$$

$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t \rightarrow (8)$$

From equations (2) & (6) we see that $S_0(t) = \frac{1}{2} S_i$

|||ly from (3) & (7) we have

$$N_0 = \left(\frac{1}{4}\right) N_i = \frac{\eta f_m}{2} \text{. Hence}$$

$$\left(\frac{S_0}{N_0}\right)_{DSB\,SC} = \frac{S_I}{\eta f_m} = d \rightarrow (9)$$

$$\frac{S_0}{N_0} = \frac{S_i}{\eta f_m} = d \rightarrow (10)$$

Comparing (9) & (10) we see that, for a given demodulator i/p signal power and identical noise conditions in the channel, the noise performance of a DSB-SC s/m is identical to that of the reference baseband s/m

# Noise Calculations in SSB Modulations:-

To obtain the noise performance of an SSB-SC s/m draw the model for the SSB receiver. An SSB-SC signal is demodulated similar to DSB-SC, by using a coherent (synchronous) detector. Hence, the model of an SSB receiver in the presence of additive channel noise will be the same as that for the DSB-SC receiver with the difference that the i/p BPF will now have a bandwidth of $\omega_m$ corresponding to the single sideband & not $2\omega_m$ as in the case of DSB-SC. Hence, the passband of the receiver i/p BPF for SSB-SC demodulation will be $\omega_c - \omega_m$ to $\omega_c$ for LSB-SSC & up to $\omega_c + \omega_m$ USB-SSC. figure 1 shows the SSB-SC receiver for noise calculation

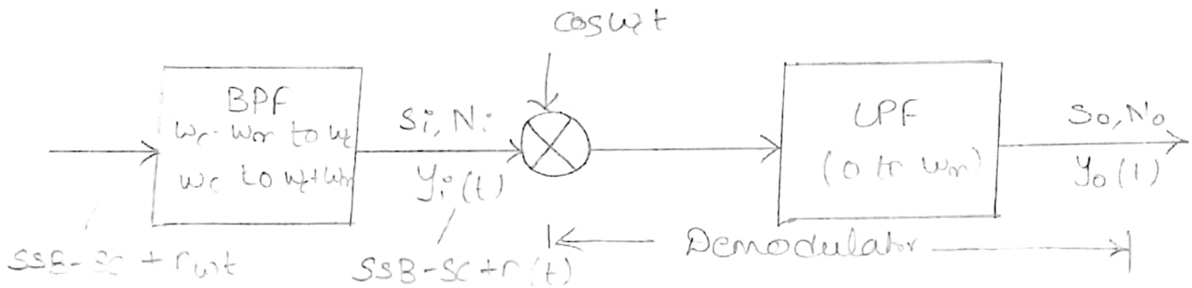



figure : SSB-SC demodulation

To i/p to the SSB-SC receiver is the SSB-SC plus additive white noise. The s/p passes through the (ideal) unity gain bandpass filter unchanged whereas the white noise, upon filtering yields bandpass noise. Hence, the i/p to the demodulator is:

$$y_i(t) = S_i(t) + n_i(t) = A_c\left[m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t\right] + n(t) \rightarrow (1)$$

The i/p signal power is the mean square value of

$S_i(t) = A_c\left[m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t\right]$ & is given by

$$S_i = \overline{\left\{A_c\left[m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t\right]\right\}^2}$$

This expression can be simplified by recalling that a function & its Hilbert transform are orthogonal. If $m(t)=0$ then $\overline{m(t)\hat{m}(t)} = 0$. Thus,

$$S_i = \left(\frac{A_c^2}{2}\right)\left[\overline{m^2(t) + \hat{m}^2(t)}\right]$$

Another useful property is that a function & its Hilbert transform have equal power. Thus is $\overline{\hat{m}^2(t)} = \overline{m^2(t)}$. Hence, the demodulator i/p signal power is given by,

$$S_i = A_c^2\, \overline{m^2(t)} \longrightarrow (1)$$

In order to compute the demodulator i/p noise power, we need to find out the area under the bandpass noise PSD for SSB-SC lower sideband (LSB) T^xion, the bandpass noise PSD is shown in figure 2.

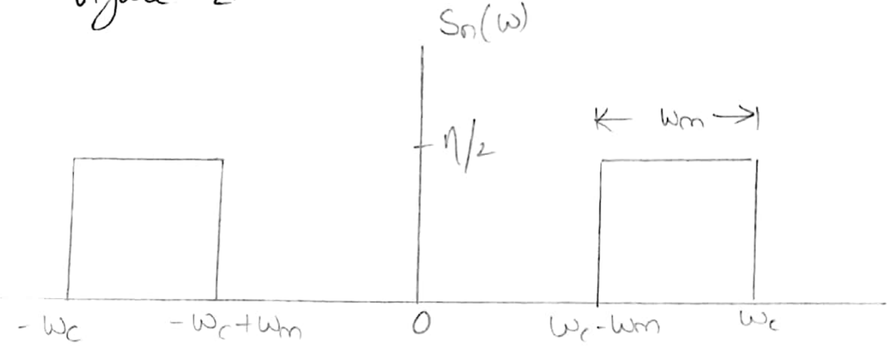



fig: PSD of bandpass white noise at SSB demodulation i/p

The demodulator i/p noise power

$$N_i = \overline{n^2(t)} = \text{Area under the PSD}$$

$$= 2\int_{f_c-f_m}^{f_c} \frac{\eta}{2}\, df = \eta f_m \longrightarrow (3)$$

In order to find out the o/p signal & noise powers, we have to obtain the expression for the demodulator o/p, $y_o(t)$

$$y_o(t) = \text{LPF}\{y_i(t) \times \cos\omega_c t\}$$

$$= \text{LPF}\{([A_c m(t)\cos\omega_c t \pm A_c \hat{m}(t)\sin\omega_c t] + n(t)) \times \cos\omega_c t\}$$

By using the quadrature representation for n(t), expanding & then eliminating the terms rejected by the LPF, we obtain:

$$y_o(t) = s_o(t) + n_o(t) = \left(\frac{A_c}{2}\right) m(t) + \frac{n_c(t)}{2} \longrightarrow (4)$$

Hence, the demodulator o/p signal power is the mean square value of $s_o(t) = \left(\frac{A_c}{2}\right) m(t)$ and is given by:

$$S_o = \left[\overline{\left(\frac{A_c}{2}\right) m(t)}\right]^2 = \frac{A_c^2}{4} \overline{m^2(t)} = \frac{1}{4} S_i \longrightarrow (5)$$

The o/p noise power is given by

$$S_o = \overline{n_o^2(t)} = \left[\overline{\left(\frac{n_c(t)}{2}\right)}\right]^2 = \frac{1}{4} \overline{n_c^2(t)} = \frac{1}{4} \overline{n_c^2(t)} = \frac{1}{4} \eta f_m \rightarrow (6)$$

From Eq (4) & (5), we obtain the o/p SNR for SSB-SC as:

$$\left(\frac{S_o}{N_o}\right)_{SSB-SC} = \frac{S_i}{\eta f_m} = d \longrightarrow (7)$$

Hence, we see that, for the same demodulator i/p signal power, the noise performance of SSB-SC system as well as DSB-SC system are identical and equal to that of the reference baseband system.

## Synchronous Detection

The model of the AM receiver using synchronous detection is the same as that used for the detection of DSB-SC signal as shown in figure

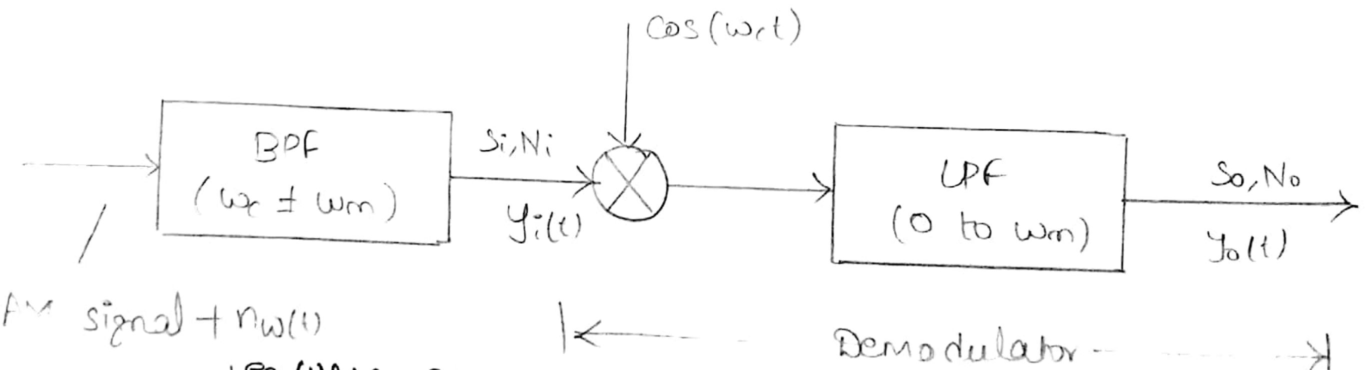



Fig.(i) AM receiver using synchronous detection

Scanned by CamScanner

The i/p to the receiver is the conventional AM signal plus noise. $y_i(t)$, the o/p of the BPF is the signal plus bandpass noise:

$$y_i(t) = s_i(t) + n_i(t) = A_c\left[1 + K_a m(t)\right]\cos\omega_c t + n(t) \longrightarrow (1)$$

Hence, the demodulator i/p signal power:

$$S_i = \overline{s_i^2(t)} = \overline{\{A_c[[1+K_a m(t)]\cos\omega_c t + n(t)]\}^2} =$$

$$= \overline{A_c^2\left[1+K_a m(t)\right]^2}$$

$$= \left(\frac{A_c^2}{2}\right)\left[1 + K_a^2\,\overline{m^2(t)} + 2K_a\,\overline{m(t)}\right]$$

$$= \frac{A_c^2}{2}\left[1 + K_a^2\,\overline{m^2(t)}\right] \longrightarrow (2)$$

Since $m(t)$ is assumed to have zero mean

$$N_i = \overline{n^2(t)} = \text{Area under the PSD from } \omega_c - \omega_m \text{ to } \omega_c + \omega_m$$

$$= 2\left(\frac{\eta}{2}\right)(2f_m) = 2\eta f_m \longrightarrow (3)$$

In order to find out the o/p signal & noise powers, we have to obtain the expression for the demodulator o/p $y_o(t)$. from figure (1)

$$y_o(t) = \text{LPF}\{y_i(t) \times \cos\omega_c t\}$$

$$= \text{LPF}\{(A_c[1+K_a m(t)]\cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t) \times \cos\omega_c t\}$$

By expanding & discarding the terms rejected by the LPF, we obtain:

$$y_o(t) = \left(\frac{A_c}{2}\right) + \left(\frac{A_c}{2}\right)K_a m(t) + \frac{n_c(t)}{2} \longrightarrow (4)$$

The first term on the right-hand side of Eq (5) is a DC term which can be removed by using a blocking capacitor. The desired s/n component is $y_o(t)$ is $s_o(t) = \left(\frac{A_c}{2}\right)K_a m(t)$ and the noise component in the demodulator o/p is

$n_o(t) = \frac{n_c(t)}{2}$. Therefore, the o/p signal & noise power are:

$$S_o = \left[\overline{\left(\frac{A_c}{2}\right) K_a m(t)}\right]^2 = \frac{A_c}{2} K_a^2 \overline{m^2(t)}$$

$$N_o = \left[\overline{\frac{n_c(t)}{2}}\right]^2 = \frac{\overline{n_c^2(t)}}{4} = \frac{\overline{n^2(t)}}{4} = \frac{N_i}{4} \longrightarrow (5)$$

Hence, the o/p SNR power for a conventional AM s/m using synchronous detection is:

$$\frac{S_o}{N_o} = \left[\frac{A_c^2}{4} K_a^2 \overline{m^2(t)}\right] \Big/ \left[\frac{N_i}{4}\right] \longrightarrow (6)$$

Eq (6) can be expressed in terms of the i/p signal power, $S_i$, as:

$$\frac{S_o}{N_o} = \frac{K_a^2 \overline{m^2(t)}}{1 + K_a^2 \overline{m^2(t)}} \left(\frac{S_i}{\eta f_m}\right) = \frac{K_a^2 \overline{m^2(t)}}{1 + K_a^2 \overline{m^2(t)}} \times \alpha \longrightarrow (7)$$

The ratio $\left[\frac{K_a^2 \overline{m^2(t)}}{1 + K_a^2 \overline{m^2(t)}}\right]$ is called the AM transmission efficiency. It is the ratio of the sideband power to the total power in the transmitted signal. Since this ratio is less than unity, Eq (7) that the SNR in conventional AM using synchronous detection is always smaller than $\alpha$ which is the o/p SNR in the case of the baseband systems.

In order to get a feel for the above result, let us consider an AM system operating with a $K_a$ of 0.8 & having a normalized message power $\overline{m^2(t)} = 0.1$ which is the typical value for speech signals. Substituting the values of $K_a$ and $\overline{m^2(t)}$ in Eq (7), we obtain

$$\frac{S_o}{N_o} = \frac{0.8^2(0.1)}{1 + 0.8^2(0.1)} \alpha = 0.06\alpha$$

$$= A_c + A_c k_a m(t) + n_c(t) \longrightarrow (11)$$

The o/p of the envelope detector is given by Eq (11)
The first term $A_c$ on the right-hand side is a DC term which can be blocked by a capacitor. The second term $A_c k_a m(t)$ is the useful signal component in the demodulator o/p & the third term $n_c(t)$ represents the noise component. Hence, the o/p signal & noise powers are given by

$$S_o = \overline{[A_c k_a m(t)]^2} = A_c^2 k_a^2 \overline{m(t)} \text{ and}$$

$$N_o = \overline{n_c^2(t)} = \overline{n^2(t)} = 2\eta f_m \longrightarrow (12, 13)$$

Therefore, the o/p SNR is given by

$$\frac{S_o}{N_o} = \frac{A_c^2 k_a^2 \overline{m(t)}}{2\eta f_m} \longrightarrow (14)$$

The right-hand side of Eq (14) can be expressed in terms of i/p signal power

$$S_i = \left(\frac{A_c^2}{2}\right)\left[1 + k_a^2 \overline{m^2(t)}\right] \text{ as follows:}$$

$$\frac{S_o}{N_o} = \frac{k_a^2 \overline{m^2(t)}}{1 + k_a^2 \overline{m(t)}} \left(\frac{S_i}{\eta f_m}\right) = \frac{k_a^2 \overline{m^2(t)}}{1 + k_a^2 \overline{m(t)}} \, d \longrightarrow (15)$$

This result is identical to Eq (8) obtained for synchronous detection of AM. Hence, for low noise, the performance of the envelope detector is identical to that of the synchronous detector.

High Noise (Low SNR):-

For $A_c[1 + k_a m(t)] \ll n_c(t), n_s(t)$, Eq (11) reduces to:

$$E(t) = \sqrt{n_c^2(t) + 2A_c[1 + k_a m(t)]n_c(t) + n_s^2(t)}$$

$$= \left[\sqrt{n_c^2(t) + n_s^2(t)}\right] \times \sqrt{1 + \frac{2A_c[1+k_a m(t)]}{n_c^2(t) + n_s^2(t)}} \rightarrow (16)$$

using the approximation $\sqrt{1+x} = \left(1+\frac{x}{2}\right)$ for $|x| << 1$ the second square-root on the right hand side Eq (16) reduces to:

$$E(t) = \left[\sqrt{n_c^2(t) + n_s^2(t)}\right] \times \left[1 + \frac{A_c[1+k_a m(t)]}{n_c^2(t) + n_s^2(t)}\right]$$

Clearly, at the demodulator o/p, the signal & noise components are no longer additive. Since the signal component is multiplied by noise, it can no longer be recovered. The Phenomenon of distortion of the message signal at low i/p SNR is called the threshold effect.

Noise in Phase Modulated systems:-

If an FM demodulator is used to receive a PM signal, it yields the o/p $\frac{dm(t)}{dt}$. By passing this o/p thru an ideal integrator, we obtain the desired signal $m(t)$ as shown in figure.

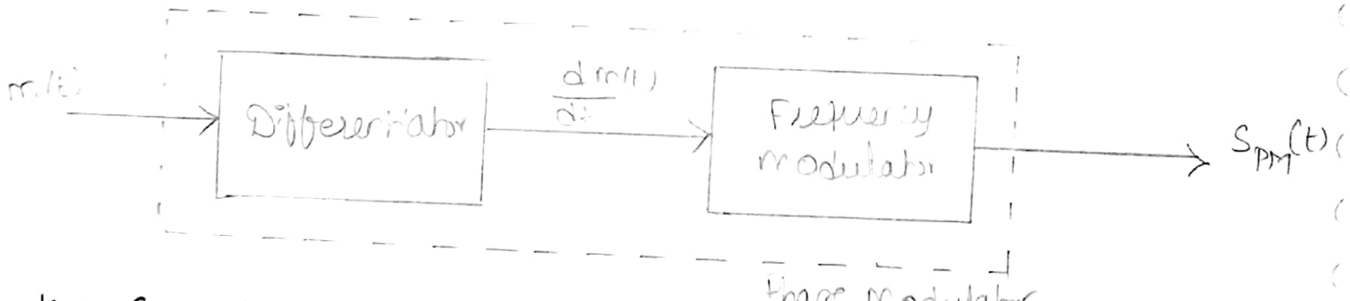



fig:1 Generation of phase modulated s/n using frequency modulator

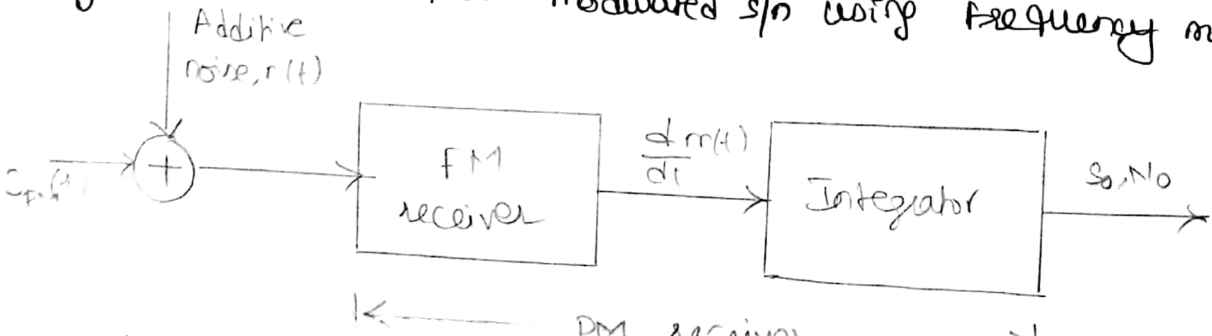



fig: 2 PM demodulator using FM receiver

$10\log_{10}(0.06) = -12\,\text{dB}$. Hence, this system is about 12dB inferior to the ideal ~~same~~ system requiring the same BW.

Envelope Detection:-

~~We recall the~~

The noise calculation model of an AM receiver using envelope detection is shown in figure (2)

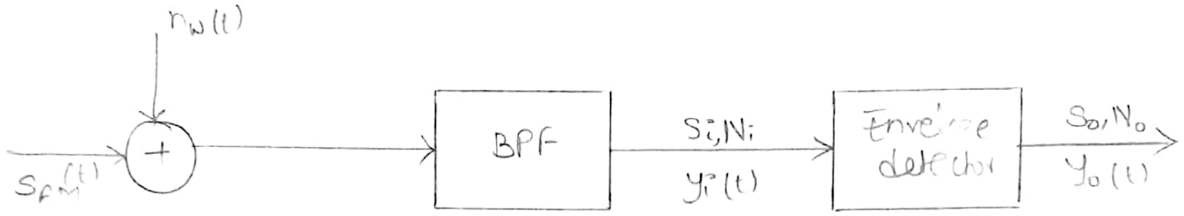



fig: 2 AM Receiver using Envelope Detector

The only difference b/w this model & the earlier one using synchronous detection is in the demodulator. Hence, the expressions for $y_i(t)$, $S_i$ and $N_i$ are the same as before & are given below for convenience:

$$y_i(t) = A_c[1 + k_a m(t)]\cos\omega_c t + n(t)$$

$$S_i = \left(\frac{A_c^2}{2}\right)\left[1 + k_a^2\,\overline{m^2(t)}\right] \quad ; \quad N_i = 2\eta f_m$$

and

To obtain the signal & noise powers at the demodulator o/p, we obtain the envelope of $y_i(t)$ as follows:

$$\left.\begin{aligned} y_i(t) &= A_c[1 + k_a m(t)]\cos\omega_c t + n(t) \\ &= A_c[1 + k_a m(t)]\cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t \\ &= \{A_c[1 + k_a m(t)]\cos\omega_c t + n_c(t)\}\cos\omega_c t - n_s(t)\sin\omega_c t \end{aligned}\right\} \rightarrow (8)$$

Equation (8) is the form of $A(t)\cos\theta(t) + B(t)\sin\theta(t)$, where

$$A(t) = A_c[1 + k_a m(t)]\cos\omega_c t + n_c(t)$$

$$B(t) = -n_s(t) \quad \& \quad \theta(t) = \omega_c t$$

Hence $y_i(t)$ can be written in terms of the envelope and phase angle as $y_i(t) = E(t)\cos[w_c t + \theta(t)]$, where the envelope $E(t) = \sqrt{A(t)^2 + B(t)^2}$ & the phase angle is $\phi(t) = \tan^{-1}\left[\frac{B(t)}{A(t)}\right]$. The envelope detector is insensitive to the carrier phase & detectors only the envelope of the i/p signal. Hence, the ideal envelope detector o/p is:

$$E(t) = \sqrt{\{A_c[1+k_a m(t)] + n(t)\}^2 + \{-n_s(t)\}^2}$$

$$= \sqrt{A_c[1+k_a m(t)]^2 + n_c^2(t) + 2A_c[1+k_a m(t)]n_c(t) + n_s^2(t)} \rightarrow (9)$$

In order to interpret Eq (9) let us consider two kinds of noise situation: low noise & high noise. For low noise in the demodulator i/p $y_i(t)$, the signal component $s_i(t)$, is large compared to the noise component $n_i(t)$. That is $A_c[1+k_a m(t)] >> |n_s(t)|$. For high noise, the noise component in $y_i(t)$ is large compared to the signal component. That is;

$A_c[1+k_a m(t)] << |n_c(t)|, |n_s(t)|$. Let us consider two cases separately.

## Low Noise (High SNR) :-

For $A_c[1+k_a m(t)] >> |n_c(t)|, |n_s(t)|$ Eq (9) reduces to:

$$E(t) = \sqrt{A_c[1+k_a m(t)]^2 + n_c^2(t) + 2A_c[1+k_a m(t)]n_c(t)}$$

$$= A_c[1+k_a m(t)]\sqrt{1 + \frac{2n_c(t)}{A_c[1+k_a m(t)]}} \longrightarrow (10)$$

since $\sqrt{1=x} = \left(1+\frac{x}{2}\right)$ for $|x| << 1$ Eq (10) reduces to:

$$E(t) = A_c[1+k_a m(t)]\left[1 + \frac{n_c(t)}{A_c[1+k_a m(t)]}\right]$$

The expression for the PM signal

$$S_{PM}(t) = A_c \cos[\omega_c + K_p m(t)] \longrightarrow (1)$$

Hence, the normalized i/p signal Power,

$$S_i = \frac{A_c^2}{2} \longrightarrow (2)$$

The i/p noise power is the noise power within a BW of & is given by

$$N_i = 2\left(\frac{\eta}{2}\right)(2\Delta f) = 2\eta\Delta f \longrightarrow (3)$$

The instantaneous frequency of the PM signal is

$$\omega_i(t) = \frac{d}{dt}[\omega_i(t) + K_p m(t)] = \omega_i + K_p \frac{dm(t)}{dt} \rightarrow (4)$$

Recall the FM receiver o/p is proportional to the instantaneo-us frequency, Hence, the FM receiver o/p $= \alpha\omega_c + \alpha K_p \frac{dm(t)}{dt}$

The first term above is a DC component. Hence, the s/n component at the o/p of the FM receiver is $\alpha K_p \frac{dm(t)}{dt}$. This component is integrated to obtain the FM receiver o/p $S_o(t) = \alpha K_p m(t)$
Hence, the PM receiver o/p s/n power is:

$$S_o(t) = \alpha^2 K_p^2 \overline{m^2(t)} \longrightarrow (5)$$

$n_d(t) = -\left(\frac{\alpha}{A_c}\right)\frac{dn_s(t)}{dt}$ that, for the small-noise case the noise at the o/p of the FM receiver is $-\left(\frac{\alpha}{A_c}\right)\left(\frac{dn_s(t)}{dt}\right)$

upon integration, this gives the noise at the o/p of the PM recei

$$n_o(t) = -\left(\frac{\alpha}{A_c}\right) n_s(t) \longrightarrow (6)$$

Hence, the PSD of this o/p noise is given by:

$$S_{n_o}(\omega) = -\left(\frac{\alpha}{A_c}\right) S_{n_o}(\omega)$$

$$S_{nd}(\omega) = \begin{cases} \frac{\alpha^2}{A_c^2}\left[S_{no}(\omega-\omega_c) + S_{no}(\omega+\omega_c)\right] & \text{for } |\omega| < \omega_m \\ 0, & \text{otherwise} \end{cases}$$

Assuming white noise (PSD), we obtain

$$S_{no}(\omega) = \begin{cases} \frac{\alpha^2 \eta}{A_c^2} & \text{for } |\omega| < \omega_m \\ 0 & \text{otherwise} \end{cases} \longrightarrow (7)$$

From Eq (7) that the o/p noise for PM has a uniform PSD. This is in contrast to FM where the o/p noise PSD has a parabolic spectrum. Hence, there is no need for pre-emphasis & de-emphasis in PM systems.

The o/p noise power is obtained as the area under the o/p noise PSD:

$$N_o = 2\int_0^{f_m} \frac{\alpha^2 \eta}{A_c^2}\, df = \frac{2\alpha^2 \eta f_m}{2\eta f_m} \longrightarrow (8)$$

using (5) & (8), the o/p SNR for the PM receiver is given by:

$$\left(\frac{S_o}{N_o}\right)_{PM} = \frac{K_p^2 A_c^2 \overline{m^2(t)}}{2\eta f_m} \longrightarrow (9)$$

## Threshold Effect in Angle Modulated systems:-

In angle modulation, the SNR at the demodulator i/p is high for noise analysis. using this, the signal & noise components at the demodulator o/p are additive. The concept of high signal to noise ratio at the demodulator o/p is used in the analysis of non linear modulation systems. Due to non-linearity, it is not compulsory that the additive s/n and noise component at the i/p of the detector results the additive signal & noise components at the o/p of the detector.

So, the concept of "signal to noise ratio at the i/p is high" is not at all correct. At low SNR, signal & noise components, are intermixed that one cannot recognize the S/n from noise. Therefore, SNR is not defined as a measure of performance.

Threshold Effect

The Effect due to which the signal cannot be distinguishable from the noise particularly at low signal to noise ratio is called "threshold Effect" (or) "mutilation".

Due to threshold effect there exists a signal to noise ratio at the i/p of demodulator which is known as "Threshold SNR", beyond which s/n mutilation occurs. The existence of the threshold Effect places an upper limit on the trade-off b/w bandwidth & power in an FM system. The limit which Explains the above trade-off is modulation index β.

The approximate relation b/w SNR of a baseband s/m & modulation index at thresholding is,

$$\left(\frac{S}{N}\right)_{b,th} = 20(\beta+1) \longrightarrow (1)$$

We can calculate the max value of β for a given received power, such that the s/m works above threshold.

For the given BW '$B_T$' we can also calculate appropriate value of 'β' using Carson's rule

$$\text{i.e } B_T = 2(\beta+1)W \longrightarrow (2)$$

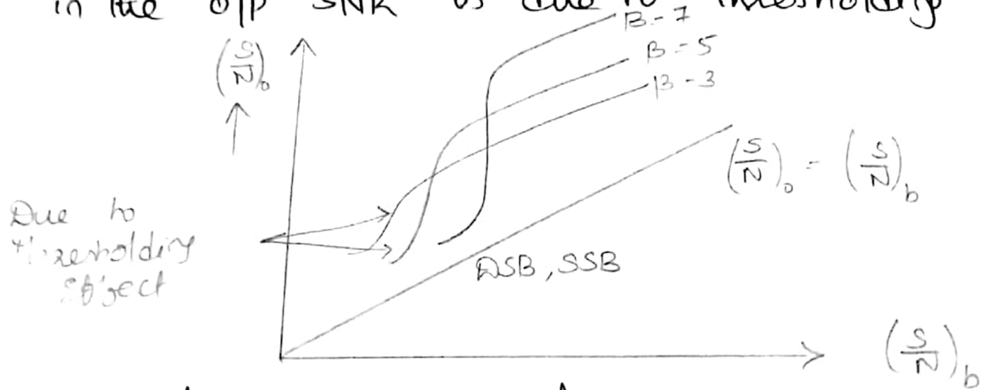
Then, using equation (1) we can calculate the required minimum receiver power.

The factors which will limit the value of modulation index are,

1. Limitation on channel BW, which affects β thru Carson's rule

2. The limitation on the received power which affects the value of $\beta$, less than what is obtained in equation (1)

Figure shows the graph b/w the SNR in an FM s/m and SNR of an baseband s/m. The graph is plotted for different values of $\beta$ as shown in the figure. The sudden drop in the o/p SNR is due to thresholding effect




consider a sinusoidal message for which,

$$\frac{P_m}{(\max |M(t)|^2)} = \frac{1}{2} \longrightarrow (3)$$

for the above message s/n case

$$\left(\frac{S}{N}\right)_o = \frac{3}{2}\beta^2 \left(\frac{S}{N}\right)_b \longrightarrow (4)$$

for example, $\beta = 7$ the above relation yields

$$\left.\left(\frac{S}{N}\right)_o\right|_{dB} = 18.66 + \left.\left(\frac{S}{N}\right)_b\right|_{dB}$$

$$\left(\frac{S}{N}\right)_{b,th} = 20(\beta+1) = 20 \times 8 = 160 = 22.04\,dB$$

for $\beta = 3$, the above relation yields,

$$\left.\left(\frac{S}{N}\right)_o\right|_{dB} = 11.3 + \left.\left(\frac{S}{N}\right)_b\right|_{dB}$$

$$\left(\frac{S}{N}\right)_{b,th} = 20(\beta+1) = 20 \times 4 = 80 = 19.03\,dB$$

From the above computation it is clear that, for example $\left(\frac{S}{N}\right)_b = 20dB$ regardless of the available BW.

## The Threshold Effect in Frequency Modulation (FM):-

The FM signal at the demodulator i/p can be given as

$$S_i(t) = A_c \cos[\omega_c t + \phi_t + n_i t] \longrightarrow ①$$

Where

$$\phi(t) = 2\pi k_f \int_0^t m(t)\,dt$$

$$n_i(t) = n_c(t)\cos(\omega_c t - n_s(t)\sin(\omega_c t))$$

$$= r(t)\cos[\omega_c t + \theta(t)]$$

$$\omega_c = 2\pi f_c = \text{Angular carrier frequency}$$

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)} \longrightarrow ②$$

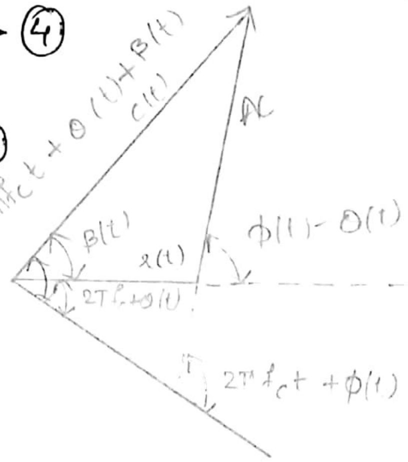
$$\theta(t) = \tan^{-1}\left[\frac{n_s(t)}{n_c(t)}\right]$$

Substituting equation ② in equation ①, we get

$$S_i(t) = A_c \cos[\omega_c t + \phi(t) + r(t)\cos[2\pi f_c t + \theta(t)]] \longrightarrow ③$$

The phasor diagram of threshold effect in FM is shown in fig below

$$S_i(t) = e(t)\cos[2\pi f_c t + \phi(t) + \beta(t)] \rightarrow ④$$




Where

$$\beta(t) = \tan^{-1}\left[\frac{A_c \sin[\phi(t) - \theta(t)]}{r(t) + A_c\cos[\phi(t) - \theta(t)]}\right] \rightarrow ⑤$$

for a large noise, $r(t) >> A_c$ Eq ⑤ can be written as

$$\beta(t) \cong \tan^{-1}\left\{\frac{A_c \sin[\phi(t) - \theta(t)]}{r(t)}\right\}$$

$$\cong \frac{A_c}{r(t)}\sin[\phi(t) - \theta(t)] \longrightarrow ⑥$$

The FM demodulator o/p is given by

$$S_d(t) = \frac{d}{dt}\{2\pi f_c t + \theta(t) + \beta(t)\}$$

$$= \frac{d}{dt}\{2\pi f_c t\} + \frac{d}{dt}\{\theta(t)\} + \frac{d}{dt}\{\beta(t)\}$$

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$= 2\pi f_c + \frac{d(\theta(t))}{dt} + \frac{d\beta(t)}{dt} \longrightarrow (7)$

The term $\frac{d\theta(t)}{dt}$ corresponds entirely to the noises. The only term which contains the information (or message) is $\frac{d\beta(t)}{dt}$.

Hence, the o/p of signal is always distorted, this gives rise to Threshold Effect

Noise Performance of PM & FM systems:-

Now, comparing the o/p signal to noise power for PM & FM systems

$$\frac{\left(\frac{S_o}{N_o}\right)_{PM}}{\left(\frac{S_o}{N_o}\right)_{FM}} = \frac{\frac{K_p^2 A_c^2 \overline{m^2(t)}}{2\eta W}}{\frac{3K_f^2 A_c^2 \overline{m^2(t)}}{2\eta W^3}} = \frac{K_p^2 W^2}{3K_f^2}$$

since $\Delta\phi = K_p$ = Modulation index in PM and

$W = f_m$, $\Delta f = K_f f_m$

$K_f = \frac{\Delta f}{f_m}$ = Modulation index in FM.

$$\frac{\left(\frac{S_o}{N_o}\right)_{PM}}{\left(\frac{S_o}{N_o}\right)_{FM}} = \frac{\beta_{PM}^2}{3\beta_{FM}^2} \quad (\text{if, } \beta_{PM} = \beta_{FM})$$

$$\boxed{\left(\frac{S_o}{N_o}\right)_{PM} = \frac{1}{3}\left(\frac{S_o}{N_o}\right)_{FM}}$$

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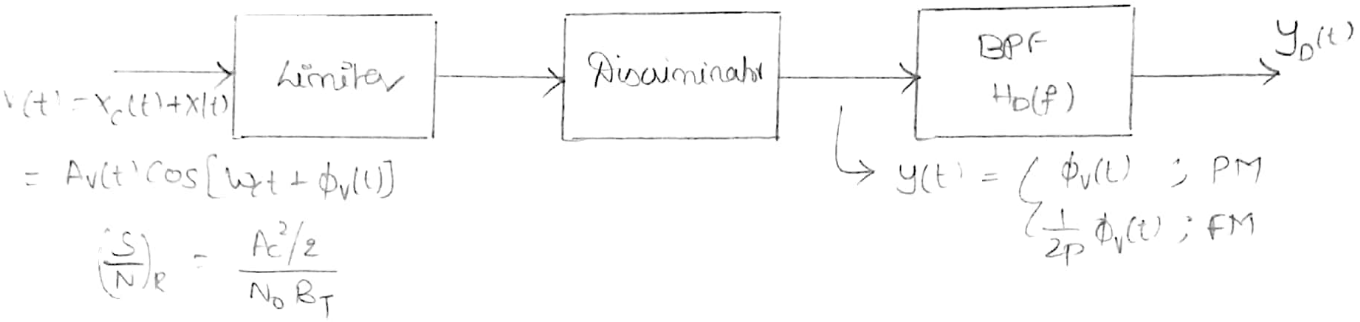
# Post detection noise in angle CW Modulation

figure (1) shows the model of the post-detector section of the exponential modulation receiver




The i/p to the detector is,

$$v(t) = x_c(t) + n(t)$$

$$v(t) = A_v(t) \cos[\omega_c t + \phi_v(t)] \rightarrow (1)$$

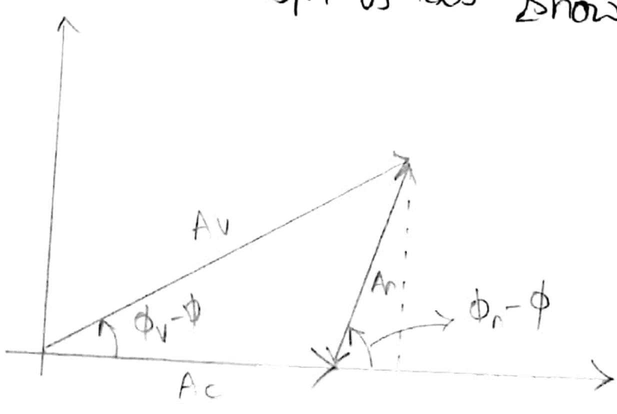
Any amplitude variation, which is represented by $A_v(t)$, will be suppresed by the limiter block.

The component $\phi_v(t)$ in equation (1) contains both signal & noise

If the noise $n(t)$ is expressed in the envelope -&- phase form then equation (1) becomes

$$v(t) = A_c \cos[\omega_c t + \phi(t)] + A_n(t) \cos[\omega_c t + \phi_n(t)] \rightarrow (2)$$

Then, the Phasor diagram of the exponential modulation - plus-noise s/n is as shown in figure (2)



from figure (2), we can write,

$$\phi_v(t) - \phi(t) = \tan^{-1} \frac{A_n(t)\sin[\phi_n(t) - \phi(t)]}{A_c + A_n(t)\cos[\phi_n(t) - \phi(t)]}$$

$$\phi_v(t) = \phi(t) + \tan^{-1} \frac{A_n(t)\sin[\phi_n(t) - \phi(t)]}{A_c + A_n(t)\cos[\phi_n(t) - \phi(t)]} \rightarrow (3)$$

T2

The first term of equation (3) i.e $\phi(t)$ is the phase of the s/n itself. The second term of the equation consists of both, the signal & the noise.

Now, as to study the post detection noise, equation (3) has to be simplified

First, consider the large signal condition

i.e $\left(\frac{S}{N}\right)_R >> 1 \Rightarrow A_c >> A_n(t)$

under such condition $\tan^{-1} x = x$

$\therefore$ equation (3) becomes

$$\phi_v(t) = \phi(t) + \frac{A_n(t)\sin[\phi_n(t) - \phi(t)]}{A_c}$$

Now replace $\phi_n(t) - \phi(t)$ by $\phi_n(t)$ in the above equation as $\phi_n(t)$ has uniform distribution over $2\pi$ radians & is same as $[\phi_n(t) - \phi(t)]$ with only the mean value shifted

$$\therefore \phi_v(t) \approx \phi(t) + \varphi(t) \rightarrow (4)$$

Here

$$\varphi(t) \triangleq \frac{A_n \sin\phi_n(t)}{A_c} = \frac{1}{\sqrt{2S_R}}\, n_q(t) \rightarrow (5)$$

$\therefore n_q = A_n \sin\phi_n$ & $S_R = \frac{A_c^2}{2}$

Now, let the detector be a phase detector

& let $\phi(t) = 0$

Then, equation (4) becomes,

$$\phi_v(t) = \psi(t)$$

Thus, the o/p of the phase detector has noise $\psi(t)$. Then, the power spectrum of the PM post detection noise is given as

$$G_\psi(f) \approx \frac{N_0}{2S_R} \Pi\left(\frac{f}{B_T}\right)$$

Which is almost flat over the range $|f| \leq \frac{B_T}{2}$, this is shown in figure (3).

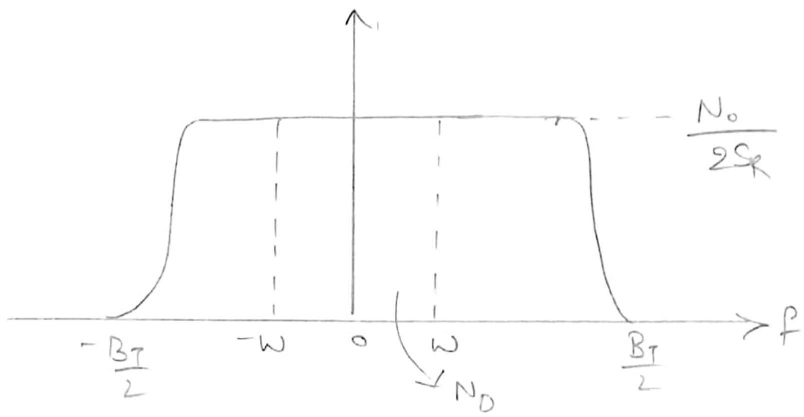



fig (3) Spectrum of the PM Post Detection Noise

As seen from figure (3), $\frac{B_T}{2}$ is beyond the message bandwidth W. In order to eliminate this out-of-band noise, a Post detection filter having transfer function $H_D(f)$ must be used.

If the filter is an ideal LPF having response $H_D(f)$, gain unity & bandwidth W, then, the o/p noise power at the destination is given as,

$$N_D = \int_{-W}^{W} G_\psi(f)\, df = \frac{N_0 W}{S_R}$$

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52, 89, 50, 48, 402, 78, 53, 79, 27, 60, 6, 35, 25
46, 7, 44, 0, 36

When the detector is a frequency detector with $\phi(t) = 0$, then from the equation (4), we get,

$$\phi_v(t) = \psi(t)$$

Let this be the i/p to the frequency detector. Then the o/p of the detector is an instantaneous frequency noise given by

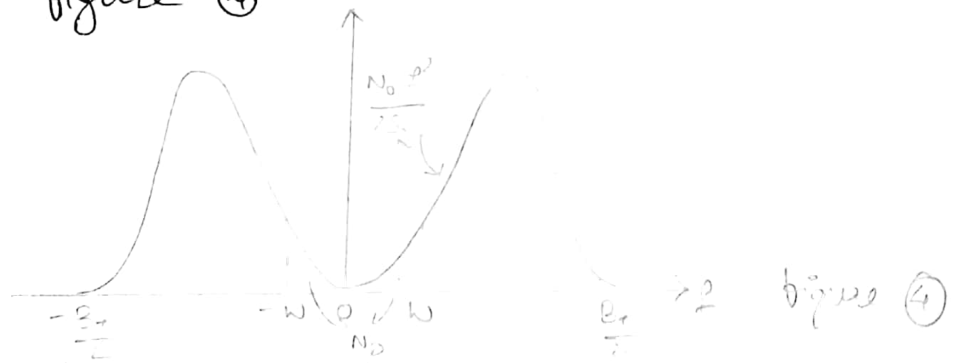
$$\xi(t) \triangleq \frac{1}{2\pi}\psi(t) = \frac{1}{2\pi\sqrt{2S_R}}\, n_q(t)$$

Then, the FM post detection noise spectrum is given

$$G_{\xi}(f) = (2\pi f)^2 \frac{1}{8\pi^2 S_R} \cdot G_{n_q}(f)$$

$$= \frac{N_0 f^2}{2S_R}\, \Pi\left(\frac{f}{B_T}\right)$$

The Spectrum of FM post detection noise is as shown in figure (4)




As shown in fig (4), the spectrum is a parabolic function having components exceeding bandwidth W, i.e., $\frac{B_T}{2} > W$ & increasing with $f^2$. Then if the post detection filter is an ideal LPF, we get the destination noise power as

$$N_D = \int_{-W}^{W} G_{\xi}(f)\, df = \frac{N_0 W^3}{3S_R}$$

## Signal to Noise Ratio:-

The ratio of signal power to the associated noise power is defined as signal to noise ratio. In other words, signal to noise ratio is defined as the ratio of signal power to noise power at the same point in the system. It is defined by S/N.

## Noise figure:-

For the comparison of receivers & amplifiers working at various impedance levels the use of the equivalent noise resistance is quite misleading. Instead of equivalent noise resistance, one another quantity known as noise figure is defined and used. sometimes this is also called noise-factor. The noise figure F is defined as the ratio of the signal-to noise power ratio supplied to the input terminals of a receiver (or) amplifier to the SNR supplied to the output or load resistor. Hence Mathematically

$$\text{Noise Figure } F = \frac{\text{Input SNR}}{\text{Output SNR}} = \frac{(SNR)_i}{(SNR)_o}$$

$$F = 1 + \frac{R_{eq}}{R_a}$$

Where $R_{eq}$ = equivalent noise resistance

$R_a$ = antenna resistance.

$$F \text{ in dB} = 10\log_{10} F$$

## Noise Temperature:

The available noise power is expressed as

$$P_n = K \cdot T \cdot B \text{ Watts}$$

This noise depends only upon the value of temperature and bandwidth. Thus, temperature is the fundamental parameter of thermal noise. Noise temperature of any white noise source either thermal or non thermal is defined as

$$T_n = \frac{P_n}{KB}$$

Where $T_n$ is a physical temperature

Noise temperature in terms of noise figure is defined as

$$T_n = T_0(F-1)$$

Where $T_0 = 17°C$ (or) $290°K$

$$P_t = K T_t B$$

$$P_t = P_1 + P_2 = KT_1B + KT_2B$$

$$T_t = T_1 + T_2$$

The equivalent noise temperature of a receiver or amplifier we have assumed that

$$R'_{eq} = R_a$$

$$\therefore F = 1 + \frac{R'_{eq}}{R_a} = 1 + \frac{T_{eq}}{T_0}$$

$$F = 1 + \frac{T_{eq}}{T_0}$$

$$\boxed{T_{eq} = T_0(F-1)}$$

## Flicker Noise:-

It is a type of electronic noise with a 1/f (or) "Pink", Power spectral density. It is therefore often referred to as 1/f noise (or) Pink noise, though those terms have wider definitions. It occurs in almost all electronic devices and can show up with a variety of other effects, such as impurities in a conductive channel, generation and recombination noise in a transistor due to base current and so on.

## Probability of error:-

In digital transmission, the number of bit errors is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or bit synchronization errors.

The bit error rate (BER) is the number of bit errors per unit time. The bit error ratio also BER is the number of bit errors divided by the total number of transferred bits during a studied time interval. Bit error ratio is a unitless performance measure often expressed as a percentage