

B.Tech II Year I Semester (R09) Supplementary Examinations June 2017

MATHEMATICS - II

(Common to AE, BT, CE & ME)

Time: 3 hours

Max. Marks: 70

Answer any FIVE questions
All questions carry equal marks

- 1 (a) Solve the system $\lambda x + y + z = 0$; $x + \lambda y + z = 0$; $x + y + \lambda z = 0$, if the system has non-zero solution only.
(b) Solve the equations: $x + y - z + t = 0$; $x - y + 2z - t = 0$; $3x + y + t = 0$.
- 2 (a) Reduce the quadratic form to canonical form by Lagrange's reduction:

$$2x_1^2 + 7x_2^2 + 5x_3^2 - 8x_1x_2 - 10x_2x_3 + 4x_1x_3$$
 And hence find rank signature of the quadratic form.
(b) Find the rank and signature of the quadratic form $x^2 - 4y^2 + 6z^2 + 2xy - 4xz + 2w^2 - 6zw$.
- 3 (a) If $f(x) = x^2$, $-l \leq x \leq l$, obtain the Fourier series and deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$
(b) Expand $f(x) = e^x$ as a Fourier series in the interval $(-l, l)$
- 4 Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & \text{if } |x| < a \\ 0, & \text{if } |x| > a > 0 \end{cases}$
 Hence show that $\int_0^\infty \frac{\sin x - x \cos x}{x^3} dx = \frac{\pi}{4}$
- 5 (a) Form the partial differential equation by eliminating the arbitrary function f from $xyz = f(x^2 + y^2 + z^2)$.
(b) Using the method of separation of variables, solve $u_{xt} = e^{-t} \cos x$ with $u(x, 0) = 0$ and $u(0, t) = 0$.
- 6 (a) Evaluate: (i) $\Delta[f(x)g(x)]$ (ii) $\Delta\left[\frac{f(x)}{g(x)}\right]$.
(b) Given $u_0 = 580$, $u_1 = 556$, $u_2 = 520$ and $u_4 = 385$ find u_3 .
- 7 (a) Fit the curve $y = ae^{bx}$ to the following data:

x	0	1	2	3	4	5	6	7	8
y	20	30	52	77	135	211	326	550	1052

(b) The following gives the velocity of a particle at time t.

t(sec.)	0	2	4	6	8	10	12
v(m/sec.)	4	6	16	34	60	94	136

 Find the distance moved by the particle in 12 sec. and also the acceleration at $t = 2$ sec.
- 8 Use Milne's method to find $y(0.8)$ and $y(1.0)$ from $y' = 1 + y^2$, $y(0) = 0$. Find the initial values $y(0.2)$, $y(0.4)$ and $y(0.6)$ from the Taylor's series method.

B.Tech II Year I Semester (R13) Supplementary Examinations June 2017

MATHEMATICS – III

(Common to EEE, ECE and EIE)

Time: 3 hours

Max. Marks: 70

PART – A

(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

- Write any two properties of beta function.
- Compute the value of $\Gamma(-1/2)$.
- Show that $p_1(x) = x$.
- State the orthogonal property of Bessels differential equation.
- Check whether $u(x, y) = \sin x \cosh y$ is harmonic function.
- Discuss about a Transformation $w = z + c$, where 'c' is complex constant.
- Evaluate $\int_C e^z dz$ where C is $|z| = 1$.
- Expand $f(z) = e^z$ in Taylor's series about $z = 1$.
- Find the residue at $z = 1$ of the function $f(z) = \frac{z^2}{(z-1)(z-2)^2}$.
- State Cauchy's residue theorem.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Prove that $\int_0^{\infty} x^{n-1} e^{-kx} dx = \frac{\Gamma n}{k^n} \quad (n > 0, k > 0)$.

(b) Prove that $\int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta = \frac{\beta(m, n)}{2}$.

OR

3 Solve in Series the equation $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$

UNIT – II

4 (a) Prove that $\frac{d}{dx} [x J_n(x) J_{n+1}(x)] = x [J_n^2(x) - J_{n+1}^2(x)]$.

(b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

OR

5 State and prove the Rodrigues formula of Legendre Polynomials.

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UNIT – III

- 6 (a) State and prove the Cauchy- Riemann equations in polar form.
 (b) If $f(z) = u + iv$ is Analytic function of z , find $f(z)$ if $2u + v = e^{2x}[(2x + y) \cos 2y + (x - 2y) \sin 2y]$

OR

- 7 (a) Find the bilinear Transformation which maps the points $z = \infty, i, 0$ into the points $w = -1, -i, 1$.
 (b) Discuss about the Transformation $w = z^2$

UNIT – IV

- 8 (a) Evaluate $\int_c \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$, where c is the circle $|z| = 1$.
 (b) Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices $-1, 1, 1 + i, -1 + i$

OR

- 9 Find the Laurent series expansion of $\frac{z^2 - 6z - 1}{(z - 1)(z - 3)(z + 2)}$ in the region $3 < |z + 2| < 5$

UNIT – V

- 10 (a) Evaluate $\int_c \frac{dz}{(z^2 + 4)^2}$, using residue theorem, where $c: |z - i| = 2$
 (b) Determine the Residue of the function $f(z) = \frac{z + 1}{z^2(z - 2)}$ at each pole.

OR

- 11 Show that $\int_0^{2\pi} \frac{\cos 2\theta}{1 - 2a \cos \theta + a^2} d\theta = \frac{2\pi a^2}{1 - a^2}$ where $a^2 < 1$
