Detection of Radar signals in Noise

INTRODUCTION

- The two basic operations performed by radar are

1. detection of the presence of reflecting Objects
2. Extraction of information from the received waveform to obtain such target data as position, velocity, and perhaps size.

- Noise ultimately limits the capability of any radar.

MATCHED FILTER RECEIVER

- A network whose frequency-response function maximizes the output peak-signal-to-mean-noise (power) ratio is called a matched filter.

- The frequency-response function, denoted H(f), expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.

- If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio.

Contd…,

- If the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy. The net result is again a lowered signal-to-noise ratio.

- Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum.

- The receiver bandwidth B should be approximately equal to the reciprocal of the pulse width τ valid only for pulsed waveforms but not to other types of waveforms.

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•The second detector and video portion of the well-designed radar super heterodyne receiver will have negligible effect on the output signal-to-noise ratio if the receiver is designed as a matched filter.

•Narrow banding is most conveniently accomplished in the IF.

•For a received waveform s(t) with a given ratio of signal energy E to noise energy *No (or)* noise power per hertz of bandwidth, North showed that the frequency-response function of the linear, time-invariant filter.

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

where $S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi ft)\, dt$ = voltage spectrum (Fourier transform) of input signal

$S^*(f)$ = complex conjugate of $S(f)$

t_1 = fixed value of time at which signal is observed to be maximum

G_a = constant equal to maximum filter gain (generally taken to be unity)

- The noise that accompanies the signal is assumed to be stationary and to have a uniform spectrum (white noise). It need not be gaussian.

- The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift exp $(-\ j2\prod ft_1)$. This phase shift varies uniformly with frequency.

Contd…,

•The frequency spectrum of the received signal may be written as an amplitude spectrum $|S(f)|$ and a phase spectrum exp $[-j\varphi_s(f)]$.

•The matchedfilter frequency-response function may similarly be written in terms of its amplitude and phase spectra $|H(f)|$ and exp $[-j\varphi_m(f)]$.

$$|H(f)| \exp[-j\phi_m(f)] = |S(f)| \exp\{j[\phi_s(f) - 2\pi f t_1]\}$$

or

$$|H(f)| = |S(f)|$$

and

$$\phi_m(f) = -\phi_s(f) + 2\pi f t_1$$

Contd…,

•The amplitude spectrum of the matched filter is the same as the amplitude spectrum of the signal, but the phase spectrum of the matched filter is the negative of the phase spectrum of the signal plus a phase shift proportional to frequency.

•The matched filter may also be specified by its impulse response h(t), which is the inverse Fourier transform of the frequency-response function.

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi f t)\, df$$

$$h(t) = G_a \int_{-\infty}^{\infty} S^*(f) \exp[-j2\pi f (t_1 - t)]\, df$$

Contd…,

- Since S*(f) = S(-f), we have

$$h(t) = G_a \int_{-\infty}^{\infty} S(f) \exp\left[j2\pi f(t_1 - t)\right] df = G_a s(t_1 - t)$$

- the impulse response of the matched filter is the image of the received waveform.

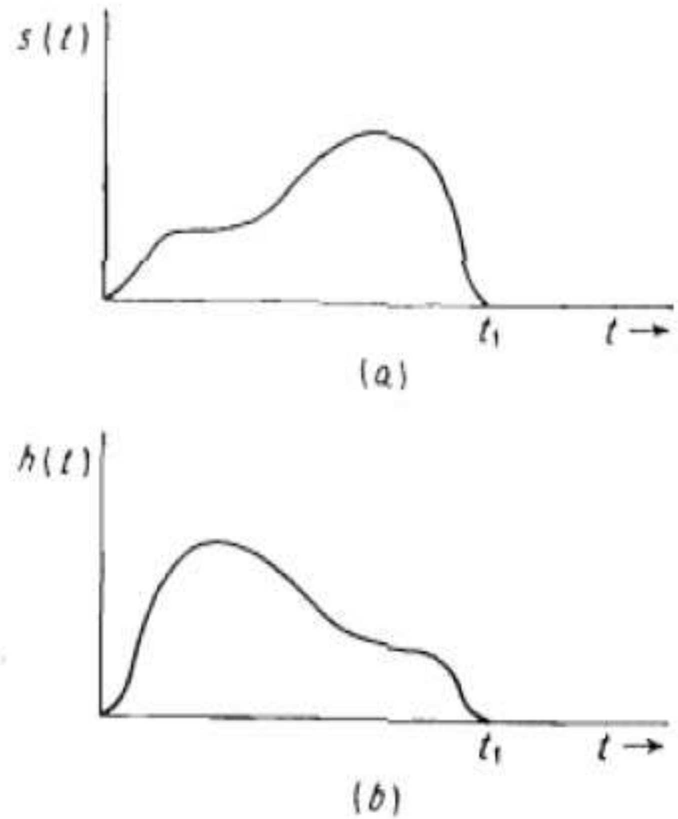

Fig.1 (a) Received waveform s(t); (b) impulse response h(t) of the matched filter.

Derivation of matched filter characteristic

We shall derive the matched-filter frequency-response function using the Schwartz inequality.

We wish to show that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

when the input noise is stationary and white (uniform spectral density). The ratio we wish to maximize is

$$R_f = \frac{|s_o(t)|^2_{\max}}{N}$$

where $|s_0(t)|_{max}$ = maximum value of output signal voltage and N = mean noise power at Receiver output

The output voltage of the filter with frequency response function H(f) is

$$|s_o(t)| = \left| \int_{-\infty}^{\infty} S(f)H(f) \exp(j2\pi ft)\, df \right|$$

where S(f) is the Fourier transform of the input (received) signal. The mean output noise power is

$$N = \frac{N_o}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df$$

where N_o is the input noise power per unit bandwidth.

Assuming that the maximum value of $|s_0(t)|^2$ occurs at t=t1 ,The ratio R_f becomes

$$R_f = \frac{\left| \int_{-\infty}^{\infty} S(f)H(f) \exp(j2\pi ft_1)\, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df}$$

Schwartz's inequality states that if P and Q are two complex functions, then

$$\int P^*P\,dx \int Q^*Q\,dx \geq \left|\int P^*Q\,dx\right|^2$$

The equality sign applies when P = kQ, where k is a constant. Letting

$$P^* = S(f)\exp(j2\pi f t_1) \quad \text{and} \quad Q = H(f)$$

and recalling that

$$\int P^*P\,dx = \int |P|^2\,dx$$

we get, on applying the Schwartz inequality to the numerator of Eq. earlier, we get

$$R_f \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |S(f)|^2\, df}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df} = \frac{\int_{-\infty}^{\infty} |S(f)|^2\, df}{\frac{N_0}{2}}$$

From Parseval's theorem,

$$\int_{-\infty}^{\infty} |S(f)|^2\, df = \int_{-\infty}^{\infty} s^2(t)\, dt = \text{signal energy} = E$$

Therefore we have

$$R_f \leq \frac{2E}{N_0}$$

The frequency-response function which maximizes the peak-signal-to-mean-noise ratio R_f may be obtained by noting that the equality sign in Eq. applies when P = kQ, or

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

where the constant k has been set equal to $1/G_a$.

The interesting property of the matched filter is that no matter what the shape of the Input signal waveform ,the maximum ratio of the peak signal power to the mean noise power is Simply twice the energy E contained in the signal divided by the noise power per hertz of Bandwidth N_0

2. Discuss the relation between the matched filter characteristics and correlation function.

- The output of the matched filter is not a replica of the input signal
- The output of the matched filter may be shown to be proportional to the input signal cross-correlated with a replica of the transmitted signal, except for the time delay t1.
- The crosscorrelation functionR(t) of two signals

y(λ) and s(λ), each defined as

$$R(t) = \int_{-\infty}^{\infty} y(\lambda) s(\lambda - t)\, d\lambda$$

- Theoutput $y_o(t)$ of a filter with impulse response $h(t)$ when the input is $y_{in}(t) = s(t) + n(t)$ is

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda) h(t - \lambda)\, d\lambda$$

- If the filter is a matched filter, then $h(\lambda) = s(t_1 - \lambda)$ and Eq. above becomes

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda) s(t_1 - t + \lambda)\, d\lambda = R(t - t_1)$$

- Thus the matched filter forms the cross correlation between the received signal corrupted by noise and a replica of the transmitted signal.
- The replica of the transmitted signal is "built in" to the matched filter via the frequency-response function.

- If the input signal $y_{in}(t)$ were the same as the signal s(t) for which the matched filter was designed, the output would be the autocorrelation function.

- The autocorrelation function of a rectangular pulse of width τ is a triangle whose base is of width 2τ.

3. Briefly explain about the efficiency of non-matched filters.

- In practice the matched filter cannot always be obtained exactly. It is appropriate, therefore, to examine the efficiency of non matched filters compared with the ideal matched filter.
- The measure of efficiency is taken as the peak signal-to noise ratio from the non matched filter divided by the peak signal-to-noise ratio (2E/No) from the matched filter.
- It can be seen that the loss in SNR incurred by use of these non-matched filters is small.

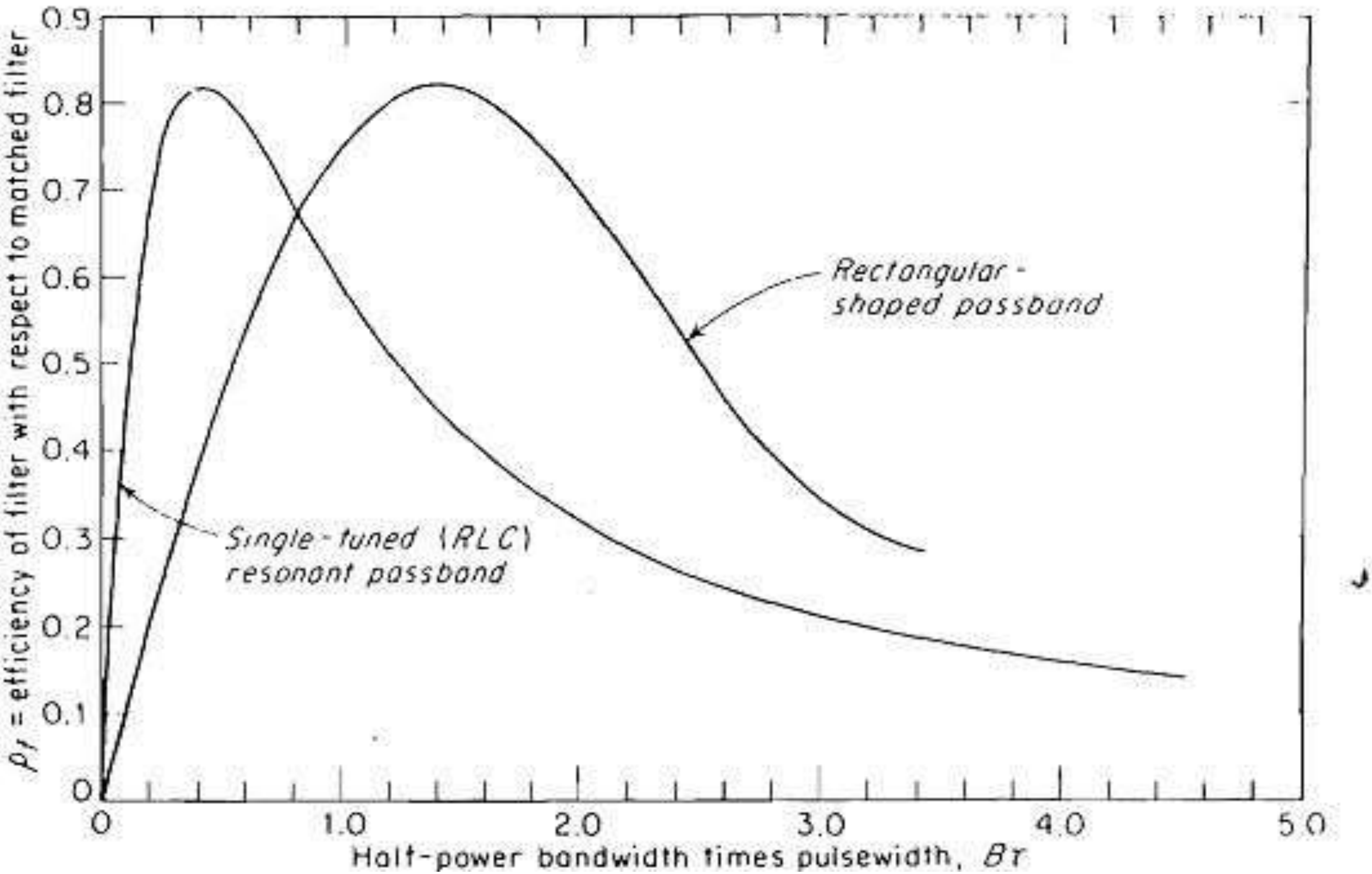

Fig.1 Efficiency, relative to a matched filter, of a single-tuned resonant filter and a rectangular shaped filter, when the input signal is a rectangular pulse of width τ. B –filter bandwidth.

Input signal	Filter	Optimum $B\tau$	Loss in SNR compared with matched filter, dB
Rectangular pulse	Rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.72	0.49
Gaussian pulse	Rectangular	0.72	0.49
Gaussian pulse	Gaussian	0.44	0 (matched)
Rectangular pulse	One-stage, single-tuned circuit	0.4	0.88
Rectangular pulse	2 cascaded single-tuned stages	0.613	0.56
Rectangular pulse	5 cascaded single-tuned stages	0.672	0.5

Table 1 Efficiency of non-matched filters compared with the matched filter

Matched filter with Non-White noise

- In the derivation of the matched-filter characteristic, the spectrum of the noise accompanying the signal was assumed to be white; that is, it was independent of frequency.
- If this assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter.
- It has been shown that if the input power spectrum of the interfering noise is given by $[N_i(f)]^2$, the frequency-response function of the filter which maximizes the output signal-to-noise ratio is

$$H(f) = \frac{G_a S^*(f) \exp(-j2\pi f t_1)}{[N_i(f)]^2}$$

- When the noise is non-white, the filter which maximizes the output signal-to-noise ratio is called the NWN (non-white noise) matched filter.
- For white noise $[N_i(f)]^2$ = constant and the NWN matched-filler frequency-response function of Eq. above reduces to that of Eq. discussed earlier in white noise. Equation above can be written as

$$H(f) = \frac{1}{N_i(f)} \times G_a \left(\frac{S(f)}{N_i(f)} \right)^* \exp(-j2\pi f t_1)$$

- This indicates that the NWN matched filter can be considered as the cascade of two filters.

- The first filter, with frequency-response function $l/N_i(f)$, acts to make the noise spectrum uniform, or white.

- It is sometimes called the whitening filter.

- The second is the matched filter when the input is white noise and a signal whose spectrum is $S(f)/Ni(f)$.